

This exam lasts 2 hours. Please silence and put away your cell phone. You are allowed 1 sheet of notes (front and back) and a calculator. Show enough work that it is clear how you arrived at your answer. Decimal answers should be rounded to 4 decimal points. Put a box around your final answer to each question. Good luck!

1. Consider the following sets.

$$A = \{0, 3, 6, 9, 12\}$$

$$B = \{0, 2, 4, 6, 8, 10, 12\}$$

(a) (6 points) True or false:

i. $0 \in A$ TRUE

iii. $0 \subseteq A$ FALSE

v. $\{4, 8, 10\} \in B$ FALSE

ii. $\emptyset \in A$ FALSE

iv. $\emptyset \subseteq A$ TRUE

vi. $\{4, 8, 10\} \subseteq B$ TRUE

(b) (2 points) What elements belong to $A \cap B$?

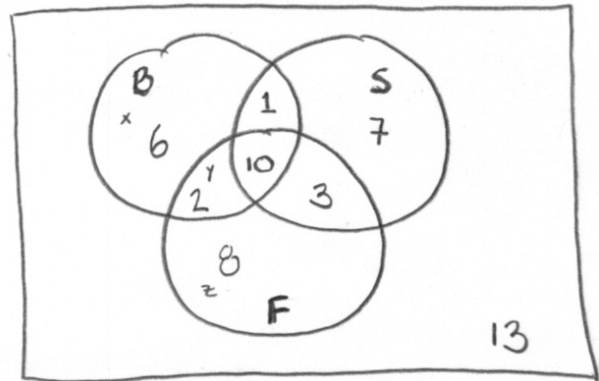
0, 6, 12

(c) (2 points) What elements belong to $A \cup B$?

0, 2, 3, 4, 6, 8, 9, 10, 12

2. A small school has a baseball team, a soccer team, and football team. However, the school has only 50 students. Thus, many students play more than one sport.

- 19 students play baseball
- ✓ • 21 students play soccer
- 23 students play football
- ✓ • 11 students play baseball and soccer
- ✓ • 13 students play soccer and football
- ✓ • 10 students play all three sports
- ✓ • 13 students play no sports



① $x + y + 1 + 10 = 19$

② $z + y + 3 + 10 = 23$

③ $x + y + z + 1 + 10 + 3 + 7 + 13 = 50$

(a) (2 points) How many students play only soccer? 7

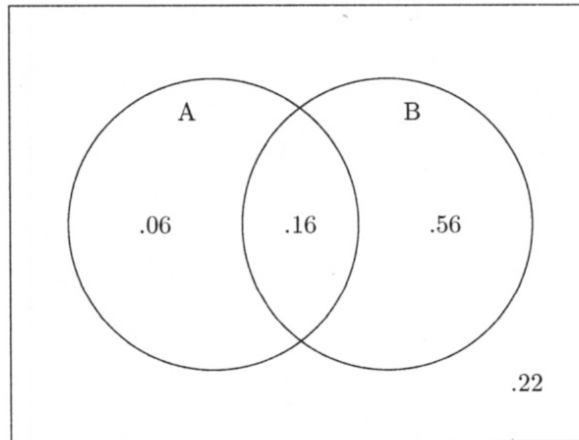
(b) (2 points) How many students play soccer and football, but not baseball? 3

(c) (2 points) How many students play baseball and football? 12

$$\begin{aligned} \textcircled{1} &\Rightarrow x + y = 8 \\ \textcircled{2} &\Rightarrow z + y = 10 \\ \textcircled{3} &\Rightarrow x + y + z = 16 \end{aligned} \quad \left. \begin{array}{l} 8 + z = 16 \\ z = 8 \end{array} \right\} \quad \begin{array}{l} \textcircled{2} \Rightarrow z + y = 10 \\ 8 + y = 10 \\ y = 2 \end{array}$$

3. An experiment can result in events A , B , both A and B , or neither with the following probabilities.
(Note: the chart and the Venn diagram are equivalent.)

	A	A'
B	.16	.56
B'	.06	.22



- (a) (3 points) Find $P(A)$.

$$P(A) = P(A \cap B) + P(A \cap B') = .16 + .06 = \boxed{.22}$$

- (b) (3 points) Find $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.16}{.72} = \boxed{.2222}$$

- (c) (3 points) Are A and B independent? Why or why not?

No. $P(A) \neq P(A|B)$.
 $.22 \neq .2222$

also, $P(A \cap B) \neq P(A)P(B)$
 $.16 \neq (.22)(.72) = .1584$

4. (a) (3 points) How many ways are there for a 12 member committee to choose a president, vice-president, and secretary?

$$3 \text{ STAGE EVENT: } 12 \cdot 11 \cdot 10 = P(12, 3) = \frac{12!}{(12-3)!} = \boxed{1320}$$

- (b) (3 points) How many ways are there for a 12 member committee to choose 5 members to serve on a subcommittee?

$$C(12, 5) = \frac{12!}{5!(12-5)!} = \boxed{792}$$

5. (6 points) Suppose you randomly select 6 animals from a group of 8 lions, 11 tigers, and 16 bears. What is the probability that you select exactly 2 lions, 2 tigers, and 2 bears?

WAYS TO CHOOSE 2 LIONS : $C(8,2)$

WAYS TO CHOOSE 2 TIGERS : $C(11,2)$

WAYS TO CHOOSE 2 BEARS : $C(16,2)$

WAYS TO CHOOSE 6 ANIMALS : $C(35,6)$

$$P(2L, 2T, 2B) = \frac{C(8,2)C(11,2)C(16,2)}{C(35,6)} = \frac{(28)(55)(120)}{1623160} \approx \boxed{.1139}$$

6. (6 points) Suppose a test for certain disease is said to be 99% accurate for the following reasons.

- If a person has the disease, the probability that they will test positive is .99.
- If a person does not have the disease, the probability that they will test negative (i.e. not positive) is .99.

If the probability that a person has the disease is only .005, find the probability that a person who tests positive for the disease actually has the disease.

LET D = PERSON HAS DISEASE

P = PERSON TESTS POSITIVE

GIVEN : $P(P|D) = .99$

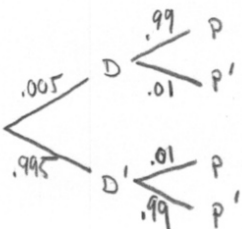
$P(P'|D') = .99 \Rightarrow P(P|D') = .01$

$P(D) = .005 \Rightarrow P(D') = .995$

FIND $P(D|P)$.

$$P(D|P) = \frac{P(D)P(P|D)}{P(P)} = \frac{P(D)P(P|D)}{P(D)P(P|D) + P(D')P(P|D')}$$

$$= \frac{(.005)(.99)}{(.005)(.99) + (.995)(.01)} \approx \boxed{.3322}$$



7. (6 points) If you roll a regular die (faces: 1, 2, 3, 4, 5, 6) 20 times, what is the probability of rolling a six exactly 4 times?

Binomial Experiment: $n = 20$ $P(x = 4) = C(20, 4) \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^{16}$
 $p = \frac{1}{6}$
 $q = \frac{5}{6}$
 $x = \# \text{ SIXES}$

$$= 4845 (0.0008) (0.0541)$$

$$= \boxed{.2022}$$

8. A raffle is being held in which 2,000 tickets are sold for \$10 each. There is 1 top prize of \$5,000, 4 middle prizes of \$500 each, and 10 lower prizes of \$100 each. All other tickets receive no prize (\$0). Let x equal the net gain/loss from buying one ticket, that is

$$x = \text{prize money} - 10.$$

- (a) (4 points) Describe the probability distribution $p(x)$ by filling in the chart below.

x	4990	490	90	-10
$p(x)$	$\frac{1}{2000}$ (.0005)	$\frac{4}{2000}$ (.0020)	$\frac{10}{2000}$ (.0050)	$\frac{1985}{2000}$ (.9925)

- (b) (4 points) Calculate the expected value $E(x)$ for x .

$$E(x) = \sum x p(x) = 4990 \left(\frac{1}{2000}\right) + 490 \left(\frac{4}{2000}\right) + 90 \left(\frac{10}{2000}\right) - 10 \left(\frac{1985}{2000}\right)$$

$$= \boxed{-6}$$

9. Consider the following set of data.

80 91 53 53 46 25 92 48 7

(a) (2 points) Compute the mean \bar{x} , and show how you arrived at your answer.

$$\bar{x} = \frac{\sum x}{n} = \frac{80 + 91 + 53 + 53 + 46 + 25 + 92 + 48 + 7}{9} = \frac{495}{9} = \boxed{55}$$

(b) (2 points) Compute the median.

7 25 46 48 $\boxed{53}$ 53 80 91 92

(c) (2 points) Compute the mode.

$\boxed{53}$

(d) (2 points) Compute the range.

$$\text{MAX} - \text{MIN} = 92 - 7 = \boxed{85}$$

10. Let z be a random variable with the standard normal probability distribution ($\mu = 0, \sigma = 1$). Use the table provided at the end of the exam or a calculator to answer the following questions.

(a) (3 points) Find the probability $P(z \leq -0.83)$

	.03
-0.8	*

$\boxed{.2033}$

(b) (3 points) Find the probability $P(z \geq 1.44)$

	.04
1.4	*

$$1 - P(z \leq 1.44) = 1 - .9251 = \boxed{.0749}$$

(c) (3 points) Determine the value z_0 such that $P(z \leq z_0) = .281$.

	.08
-0.5	.281

$\boxed{-.58}$

(d) (3 points) Determine the value z_0 such that $P(z \geq z_0) = .011$.

$$P(z \leq z_0) = 1 - .011 = .9890$$

	.09
2.2	.9890

$\boxed{2.29}$

11. Suppose that the weight of chicken eggs is normally distributed with a mean $\mu = 2.03$ oz and standard deviation of $\sigma = .24$ oz.

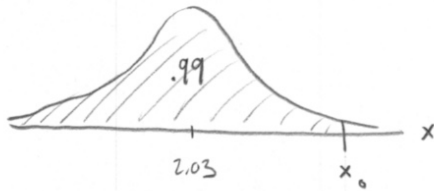
- (a) (5 points) Chicken eggs that weight between 2.15 oz and 2.35 oz are labelled "Extra Large" by the USDA. What percentage of all chicken eggs could be labelled "Extra Large"?

Let $x =$ WEIGHT OF CHICKEN EGG

$$P(2.15 \leq x \leq 2.35) = P\left(\frac{2.15 - 2.03}{.24} \leq z \leq \frac{2.35 - 2.03}{.24}\right)$$

$$\begin{aligned} &\approx P(.5 \leq z \leq 1.33) = P(z \leq 1.33) - P(z \leq .5) \\ &= .9082 - .6915 = \boxed{.2167} \end{aligned}$$

- (b) (5 points) How much does a chicken egg need to weigh in order to be heavier than 99% of all chicken eggs?

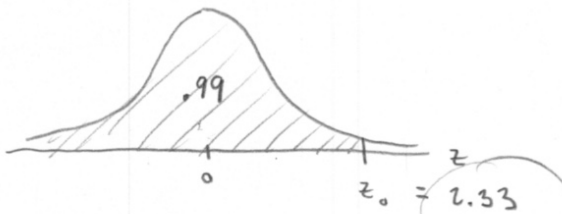


$$z_0 = \frac{x_0 - \mu}{\sigma}$$

$$2.33 = \frac{x_0 - 2.03}{.24}$$

$$x_0 = 2.03 + 2.33(.24)$$

$$= \boxed{2.5892}$$



12. (5 points) If you deposit \$1,800 into an account earning 2.3% annual interest compounded quarterly, find the account balance (i.e. *compound amount*) after 3.5 years.

$$\begin{aligned} A = Pr^n &= 1800 \left(1 + \frac{.023}{4}\right)^{3.5(4)} \\ &= 1800 (1.00575)^{14} \approx \boxed{\$ 1950.44} \end{aligned}$$

13. Suppose you currently owe your credit card company \$4,215 and you are being charged 27% annual interest compounded monthly on the amount owed.

(a) (5 points) Find the monthly *periodic payment* necessary to amortize the loan and pay off your credit card debt in 12 equal-sized monthly payments.

$$P = \frac{a(1 - r^{-n})}{r - 1}$$

P = Present Value of Annuity

$$= 4215$$

$$r = 1 + \frac{.27}{12} = 1.0225$$

$$n = 12$$

$$a = \frac{P(r - 1)}{1 - r^{-n}} = \frac{4215(.0225)}{1 - 1.0225^{-12}}$$

$$\approx \boxed{\$404.71}$$

(b) (3 points) Using your answer to part (a), what is the total amount of all payments made, and what is the total interest paid?

$$12 \times 404.71 = \boxed{\$4856.52} \quad \text{TOTAL}$$

$$4856.52 - 4215 = \boxed{\$641.52} \quad \text{INTEREST}$$