Name: * ANSWER KEY * **9/24/2019**

Math 1100-L02 Finite Mathematics **Quiz 1**

Please put away all papers and electronic devices except a calculator. Show enough work that it is clear how you arrived at your answer. Box/circle your final answers. Good luck!

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set, and let $A = \{2, 5, 7, 8\}$ and $B = \{3, 5, 6, 7\}$ be subsets of U.

   (a) (2 points) True or false: $2 \in A$ TRUE

   (b) (2 points) True or false: $A \in U$ FALSE

   (c) (2 points) True of false: $A \subseteq U$ TRUE

   (d) (3 points) Find $A \cup B$. $\{2, 3, 5, 6, 7, 8\}$

   (e) (3 points) Find $A \cap B'$.

   $A \cap B' = \{2, 5, 7, 8\} \cap \{1, 2, 4, 8, 9\}$

   $= \{2, 8\}$

   (f) (3 points) Find $A' \cup B$.

   $A' \cup B = \{1, 3, 4, 6, 9\} \cup \{3, 5, 6, 7\}$

   $= \{1, 3, 4, 5, 6, 7, 9\}$

   (g) (6 points) List all possible subsets of $A \cap B$.

   $A \cap B = \{5, 7\}$

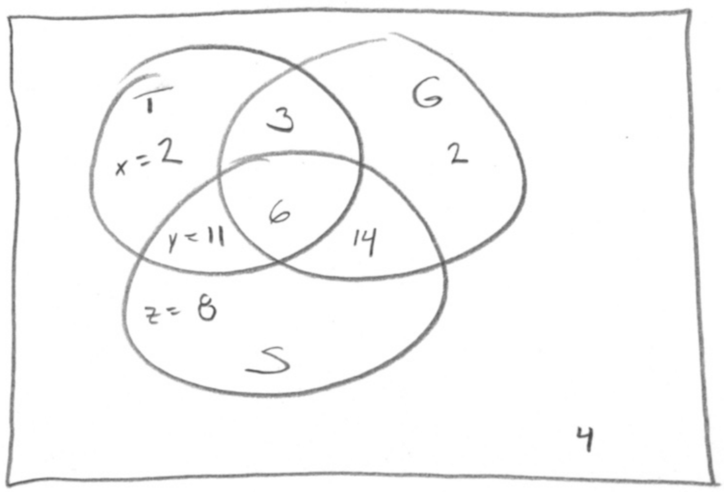
   $\{5, 7\}$,

   $\{5\}$, $\{7\}$,

   $\emptyset$

2. After a genetics experiment on 50 pea plants, the number of plants having certain characteristics was tallied, with the following results.

   - 22 were tall
   - 25 had green peas
   - 39 had smooth peas
   - 9 were tall and had green peas
   - 20 had green peas and smooth peas
   - 6 had all three characteristics
   - 4 had none of the characteristics



$x + y + 3 + 6 = 22$

$x + y = 13$

$y + z + 6 + 14 = 39$

$y + z = 19$

$x + y + z + 3 + 6 + 2 + 14 + 4 = 5$

$\underbrace{x + y} + z = 21$

$13 + z = 21$

$z = 8$

$\Rightarrow y = 11$

$\Rightarrow x = 2$

(a) (6 points) Find the number of plants that were tall and had smooth peas.

$11 + 6 = 17$

(b) (6 points) How many plants were tall and had peas that were neither smooth nor green?

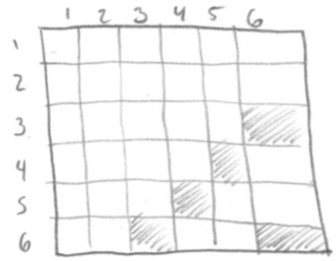
2

(c) (6 points) How many plants were not tall but had peas that were smooth and green?

14

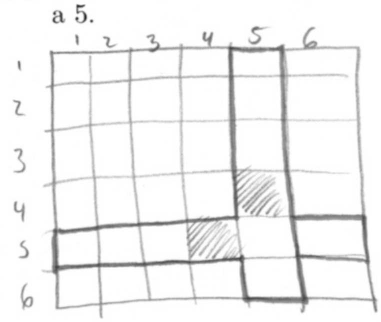
3. An experiment consists of rolling two fair dice.

(a) (8 points) Find the probability that the sum of the dice is 9 or 12.



$\frac{5}{36}$

(b) (8 points) Find the probability that the sum of the dice is 9, given that at least one the dice shows a 5.



$\frac{2}{11}$

4. You are given the following data regarding the probabilities of two events A and B.

$$P(A') = .7 \quad P(A \cup B) = .6 \quad P(B) = .5$$

(a) (8 points) Compute $P(A \cap B)$.

$P(A) = 1 - P(A') = 1 - .7 = .3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.6 = .3 + .5 - P(A \cap B) \Rightarrow P(A \cap B) = .2$

(b) (8 points) Compute $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.2}{.5} = .4$$

(c) (5 points) Are A and B independent? Why?

No. $P(A) \neq P(A|B)$

$.3 \neq .4$

or

No. $P(A \cap B) \neq P(A)P(B)$

$.2 \neq (.3)(.5)$

$.2 \neq .15$

5. (12 points) It is known that 4% of the population has a certain disease. A blood test is developed to test for the disease. A person with the disease has a 96% probability of testing positive for the disease, and a person without the disease has a 3% probability of testing positive for the disease (false positive). Find the probabilty that a person who tests positive for the disease has the disease.

LET D = PERSON HAS DISEASE

T = PERSON TESTS POSITIVE FOR DISEASE

GIVEN: $P(D) = .04$, $P(D^c) = .96$

$P(T|D) = .96$

$P(T|D^c) = .03$

FIND $P(D|T)$.

$$P(D|T) = \frac{P(D)P(T|D)}{P(D)P(T|D) + P(D^c)P(T|D^c)}$$ (BAYES' THM)

$$= \frac{(.04)(.96)}{(.04)(.96) + (.96)(.03)} = \frac{.0384}{.0672} \approx .5714$$

6. (12 points) The following table gives the proportion of US adults in each age group in 2012, as well as the proportion in each group who smoke.

Age	Proportion of Population	Proportion that Smoke
18-24 years	.131	.173
25-44 years	.345	.216
45-64 years	.345	.195
65 years and over	.179	.089

Find the probability that a randomly selected US adult smokes.

LET S_1 = 18-24 yo

S_2 = 25-44 yo

S_3 = 45-64 yo

S_4 = 65 + yo

A = SMOKES

GIVEN: $P(S_1) = .131$ $P(A|S_1) = .173$

$P(S_2) = .345$ $P(A|S_2) = .216$

$P(S_3) = .345$ $P(A|S_3) = .195$

$P(S_4) = .179$ $P(A|S_4) = .089$

LAW OF TOTAL PROBABILITY:

$$P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + P(S_3)P(A|S_3) + P(S_4)P(A|S_4)$$
$$= (.131)(.173) + (.345)(.216) + (.345)(.195) + (.179)(.089)$$
$$= .1804$$