

Please put away all papers and cell phones. You may use a calculator. Show enough work that it is clear how you arrived at your answer. Round answers to 4 decimal places when necessary. Correct answers with no work shown will not receive full credit. Box/circle your final answers. Good luck!

1. (6 points) A certain pizzeria makes custom pies. Customers choose the crust (thick or thin), the sauce (margarita, pesto, or garlic) the cheese (regular mozzarella, buffalo mozzarella, ricotta, or parmesan), and the topping (plain, pepperoni, sausage, mushroom, or anchovies). How many different custom pies is it possible to order from this pizzeria?

(MULTIPLICATION RULE) $2 \cdot 3 \cdot 4 \cdot 5 = \boxed{120}$

2. (6 points) In some countries (like Australia) and some states (like Maine) citizens vote using a *ranked voting* system. In this system, voters rank their top 3 choices of candidates for a particular office, points are awarded accordingly, and the winner is the candidate who receives the most votes. Suppose 8 candidates are running for office and you must rank your top 3 choices. How many ways are there for you to do this?

(RANK \Rightarrow ORDER MATTERS) $P(8, 3) = \boxed{336} = \frac{8!}{5!}$

3. (a) (5 points) How many different 4-letter radio station call letters are possible if the first letter must be a *K* or a *W*, and all 4 letters must be distinct?

(MULTIPLICATION RULE) $2 \cdot 25 \cdot 24 \cdot 23 = \boxed{27600}$
 (ACT: COMBINATIONS $C(2, 1) \cdot C(25, 3)$)

- (b) (5 points) How many different 4-letter radio station call letters are possible if the first letter must be a *K* or a *W*, and the last letter must be a *P*, *J*, or *X*, but letters can be repeated.

(MULTIPLICATION RULE) $2 \cdot 26 \cdot 26 \cdot 3 = \boxed{4056}$
 (A.K.A. $C(2, 1) C(26, 1) C(26, 1) C(3, 1)$)

4. (6 points) You select 3 crayons from a box of 16 crayons (all distinct colors). Assuming the order of selection does not matter, how many ways are there to do this?

$$C(16, 3) = \boxed{560} \quad \frac{16!}{3! \cdot 13!}$$

5. (8 points) A baseball team has 21 players - 16 are right-handed and 5 are left-handed. Suppose 9 players are chosen randomly to play in a particular game. What is the probability that exactly 2 of the chosen players are left handed?

$$\left. \begin{array}{l} 16 \text{ R-HANDED CHOOSE } 7 \\ 5 \text{ L-HANDED CHOOSE } 2 \end{array} \right\} \# \text{ WAYS TO DO THIS: } C(16, 7)C(5, 2) = (11440)(10) = \underline{\underline{114400}}$$

$$\text{OVERALL, 21 PLAYERS CHOOSE 9. } \# \text{ WAYS TO DO THIS: } C(21, 9) = \underline{\underline{293930}}$$

$$\therefore P(7 \text{ RH, } 2 \text{ LH}) = \frac{C(16, 7)C(5, 2)}{C(21, 9)} = \frac{114400}{293930} = \boxed{.3892}$$

$$\left(\text{Alt: } \left. \begin{array}{l} 13 \text{ R-HANDED CHOOSE } 7 \\ 8 \text{ L-HANDED CHOOSE } 2 \end{array} \right\} \rightarrow \frac{C(13, 7)C(8, 2)}{C(21, 9)} = \frac{(1716)(28)}{293930} = \boxed{.1635} \right)$$

6. (8 points) A group of 20 international students are traveling together. Eight speak only english, 6 speak only spanish, 3 speak only french, and 3 speak only german. Four students are chosen randomly to share a room at a hotel. Find the probability that at least two of the chosen students speak the same language. Hint: consider the opposite (compliment) event.

CHOOSE 1 ENGLISH, 1 SPANISH, 1 FRENCH, 1 GERMAN
SPEAKING STUDENT.

$$\# \text{ WAYS TO DO THIS: } C(8, 1)C(6, 1)C(3, 1)C(3, 1) \\ = 8 \cdot 6 \cdot 3 \cdot 3 = \underline{\underline{432}}$$

$$\text{OVERALL, } \# \text{ WAYS TO CHOOSE 4 STUDENTS FROM 20: } C(20, 4) = \underline{\underline{4845}}$$

$$\therefore P(\text{NONE THE SAME LANGUAGE}) = \frac{432}{4845} = .0892$$

$$\therefore P(\text{AT LEAST 1 THE SAME LANGUAGE}) = 1 - P(\text{NONE THE SAME LANGUAGE}) \\ = 1 - .0892 = \boxed{.9108}$$

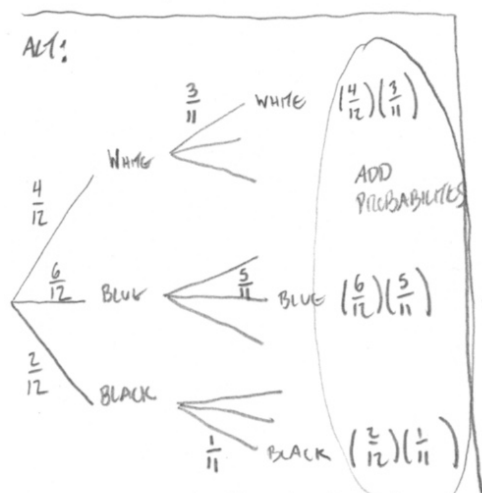
7. (8 points) Mixed in a drawer are 4 white socks, 6 blue sock, and 2 black socks. You pull out two socks, one at a time, without looking. Find the probability of getting 2 socks of the same color.

$$P(2 \text{ SAME COLOR}) = P(2 \text{ WHITE} \cup 2 \text{ BLUE} \cup 2 \text{ BLACK})$$

$$= P(2 \text{ WHITE}) + P(2 \text{ BLUE}) + P(2 \text{ BLACK}) \quad (\text{DISJUNT EVENTS})$$

$$= \frac{C(4,2) + C(6,2) + C(2,2)}{C(12,2)}$$

$$= \frac{6 + 15 + 1}{66} = \frac{22}{66} = \frac{1}{3} \approx .3333$$



8. (8 points) A die is rolled 8 times. Find the probability of rolling a six exactly twice.

(BERNOULLI PROB.)

$$n = 8$$

$$p = \frac{1}{6}$$

$$q = \frac{5}{6}$$

$x = \# \text{ TIMES } 6 \text{ IS ROLLED}$

$$P(x = k) = C(n, k) p^k q^{n-k}$$

$$P(x = 2) = C(8, 2) \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^6$$

$$= .2605$$

9. For every copy made on a particular copy machine, the probability that a jam occurs is .0031.

(a) (6 points) Suppose you make 100 copies. Find the probability that a jam does not occur.

(BERNOULLI PROB.)

$$n = 100$$

$$p = .0031$$

$$q = .9969$$

$x = \# \text{ JAMS}$

$$P(x = 0) = C(100, 0) (.0031)^0 (.9969)^{100}$$

$$= .7331$$

(b) (6 points) Suppose you make 500 copies. Find the probability that a jam does not occur.

$$\text{Now } n = 500 : P(x = 0) = C(500, 0) (.0031)^0 (.9969)^{500}$$

$$= .2117$$

10. Every time you play a Youtube video, a video ad plays. Suppose the ad is chosen randomly and 30% of ads are 5 seconds long, 45% are 15 seconds long, and 25% are 30 seconds long. Let x equal the length of the randomly selected ad in seconds.

(a) (6 points) Describe the probability distribution $p(x)$ by filling in the chart below.

x	5	15	30
$p(x)$.3	.45	.25

(b) (8 points) Calculate the expected value $E(x)$ for x .

$$\begin{aligned}
 E(x) &= x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) \\
 &= (5)(.3) + (15)(.45) + (30)(.25) \\
 &= 1.5 + 6.75 + 7.5 \\
 &= \boxed{15.75}
 \end{aligned}$$

11. A raffle is being held in which 600 tickets are sold for \$5 each. There is 1 first prize of \$1500 and there are 2 second prizes of \$500 each. All other tickets receive no prize (\$0). Let x equal the net gain/loss from buying one ticket, that is

$$x = \text{prize money} - 5.$$

(a) (6 points) Describe the probability distribution $p(x)$ by filling in the chart below.

x	1495	495	-5
$p(x)$	$\frac{1}{600}$	$\frac{2}{600}$	$\frac{597}{600}$

(b) (8 points) Calculate the expected value $E(x)$ for x .

$$\begin{aligned}
 E(x) &= (1495) \left(\frac{1}{600} \right) + (495) \left(\frac{2}{600} \right) + (-5) \left(\frac{597}{600} \right) \\
 &= \boxed{-\frac{5}{6} \approx -.8333}
 \end{aligned}$$