Name: * Answer Key * 11/26/2019

Math-1100 Finite Mathematics **Quiz 4**

Please put away all papers and electronic devices except for a calculator. Show enough work that it is clear how you arrived at your answer. Put a box/circle around your final answer to each question, rounded to 4 decimal places. Good luck!

1. (8 points) Find the simple interest earned on an investment of \$2,200 deposited into an account earning 6% annual interest for 8 months.

$$I = Pit = (2200)(.06)\left(\frac{8}{12}\right) = \boxed{\$88}$$

2. (10 points) Find the compound amount (i.e. final account balance) for an investment of \$8,000 deposited into an account earning 3% annual interest compounded monthly for 15 years.

$$A = Pr^n = (8000)\left(1 + \frac{.03}{12}\right)^{(12)(15)} = (8000)(1.0025)^{180}$$

$$= \boxed{\$12,539.45}$$

$r = 1 + \frac{i}{m}$ ← # compounds per year

$n = mt$

3. (8 points) How much money should you invest today into an account earning 3.12% annual interest compounded monthly in order for it grow to a compound amount of $30,000 in 10 years?

$$A = Pr^n \implies P = \frac{A}{r^n} = \frac{30{,}000}{\left(1 + \frac{.0312}{12}\right)^{(12)(10)}}$$

$$= \frac{30{,}000}{1.0026^{120}} = \boxed{\$\,21968.34}$$

4. (8 points) Bank A offers savings accounts that earn 2% annual interest compounded quarterly, and Bank B offers savings accounts that earn 1.9% annual interest compounded monthly. Which bank offers savings accounts with a higher effective rate? That is, which bank should you invest in?

BANK A: $i_E = \left(1 + \frac{i}{m}\right)^m - 1 = \left(1 + \frac{.02}{4}\right)^4 - 1$

$= .0202$

BANK B: $i_E = \left(1 + \frac{i}{m}\right)^m - 1 = \left(1 + \frac{.019}{12}\right)^{12} - 1$

$= .0192$

SINCE BANK A HAS A HIGHER EFFECTIVE RATE, YOU SHOULD INVEST IN $\boxed{\text{BANK A.}}$

(ALTERNATIVELY, CHOOSE ARBITRARY PRINCIPAL P AND CALCULATE COMPOUND AMOUNT AFTER SAME PERIOD OF TIME, e.g. 1 YEAR. $A = P\left(1 + \frac{i}{m}\right)^{mt}$).

5. (8 points) Consider the following geometric sequence.

$$\frac{3}{4}, \quad \frac{3}{16}, \quad \frac{3}{64}, \quad \frac{3}{256}, \quad \ldots$$

Find the sum of the first 8 terms.

THE n^{TH} TERM OF THE SEQUENCE IS GIVEN BY $a_n = ar^{n-1}$,

WHERE $a = \frac{3}{4}$ AND $r = \frac{1}{4}$.

THE SUM OF THE FIRST n TERMS IS GIVEN BY $S_n = \frac{a(1-r^n)}{1-r}$ or $\frac{a(r^n-1)}{r-1}$

$$S_8 = \frac{\cancel{\left(\frac{3}{4}\right)}\left(1-\left(\frac{1}{4}\right)^8\right)}{\cancel{1-\frac{1}{4}}} = .9999847412 \approx \boxed{1}$$

6. (12 points) Suppose you deposit \$40 at the end of every month into an account earning 4.8% annual interest compounded monthly. This is an *ordinary annuity.* Find the future value of this ordinary annuity after 4 years.

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{(40)\left[\left(1+\frac{.048}{12}\right)^{(12)(4)} - 1\right]}{\cancel{1}+\frac{.048}{12} - \cancel{1}}$$

$$= \frac{(40)(1.004^{48}-1)}{.004} = \boxed{\$2112.07}$$

7. (a) (10 points) What size periodic payments must you make at the end of each month into an account earning 3.6% annual interest compounded monthly in order to have $40,000 in the account after 18 years?

$$S_n = \frac{a(r^n - 1)}{r-1} \quad \Rightarrow \quad a = \frac{S_n(r-1)}{r^n - 1}$$

$$a = \frac{(40{,}000)(\cancel{1} + \frac{.036}{12} - \cancel{1})}{(1 + \frac{.036}{12})^{(12)(18)} - 1} = \frac{(40{,}000)(.003)}{1.003^{216} - 1}$$

$$= \boxed{\$\ 131.89}$$

(b) (8 points) How much money would you need to depoisit today into an account earning 3.6% interest compounded monthly in order to have $40,000 in the account after 18 years. That is, find the *present value* of the annuity in part (a).

$$P r^n = S_n$$

$$\Rightarrow P = \frac{S_n}{r^n} = \frac{40{,}000}{(1 + \frac{.036}{12})^{(12)(18)}}$$

$$= \frac{40{,}000}{1.003^{216}}$$

$$= \boxed{\$20{,}943.94}$$

Alt: $$P r^n = \frac{a(r^n - 1)}{r-1}$$

$$P = \frac{a(1 - r^{-n})}{r-1}$$

$$= \frac{(131.89)\left(1 - (1 + \frac{.036}{12})^{-(12)(18)}\right)}{\cancel{1} + \frac{.036}{12} - \cancel{1}}$$

$$= \frac{(131.89)(1 - 1.003^{-216})}{.003}$$

$$= \boxed{\$\ 20{,}944.19}$$

8. (a) (10 points) Find the payment necessary to amortize a 24% APR[1] loan of \$200 compounded monthly with 12 monthly payments.

$$P = \frac{a(1-r^{-n})}{r-1} \Rightarrow a = \frac{P(r-1)}{1-r^{-n}}$$

$$a = \frac{(200)\left(\cancel{1} + \frac{.24}{12} - \cancel{1}\right)}{1-\left(1+\frac{.24}{12}\right)^{-12}} = \frac{(200)(.02)}{1-1.02^{-12}}$$

$$= \boxed{\$18.91}$$

(b) (8 points) Based on your answer to part (a), what is the total amount of all payments made, and what is the total interest paid?

TOTAL PAYMENT $= 12 \times \$18.91 = \boxed{\$226.92}$

TOTAL INTEREST $= \$226.92 - \$200 = \boxed{\$26.92}$

[1] APR stands for Annual Percentage Rate, and is simply an industry term for annual interest rate, frequently used in the context of credit cards.

9. (10 points) Suppose you have just purchased your first home. Congratulations! In order to pay for it, you have taken out a loan of $300,000 at 8.4% annual interest compounded monthly, amortized with 120 equal monthly payments of $3,703.55 over the next 10 years. Complete the amortization table below for the first four monthly payments.

Payment #	Payment	Interest Payment	Applied to Principal	Principal (Balance)
0	-	-	-	$300,000
1	$3,703.55	2100.00	1603.55	298,396.45
2	$3,703.55	2088.78	1614.77	296,781.68
3	$3,703.55	2077.47	1626.08	295,155.60
4	$3,703.55	2066.09	1637.46	293,518.14

$$\text{INTEREST PAYMENT} = (\text{PRINCIPAL FROM PREVIOUS LINE})\left(\overbrace{\frac{.084}{12}}^{.007}\right)$$

$$\text{APPLIED TO PRINCIPAL} = \text{PAYMENT} - \text{INTEREST PAYMENT}$$

$$\text{PRINCIPAL} = (\text{PRINCIPAL FROM PREVIOUS LINE}) - \text{INTEREST PAYMENT}$$