§9.2 MEASURES OF VARIATION 3/23/2020

- RANGE
- DEVIATIONS FROM MEAN $\bar{x}$
- VARIANCE, STANDARD DEVIATION s
- COUNTING $\bar{x} \pm ks$
    - VALUES WITHIN k STANDARD DEV. OF MEAN
    - % OF
    - CHEBYCHEV'S THM

DEVIATION FROM THE MEAN

GIVEN DATA: 8, 12, 15, 20, 21, 14

$$\text{MEAN } \bar{x} = \frac{\sum x_i}{n} = \frac{8+12+\dots+14}{6}$$

$$= \frac{90}{6} = \underline{\underline{15}}$$

x	$x - \bar{x}$
8	-7
12	-3
15	0
20	5
21	6
14	-1
	+
	0

<u>Def</u>: GIVEN A VALUE x FROM A DATA SET WITH MEAN $\bar{x}$, THE CORRESPONDING <u>DEVIATION FROM THE MEAN</u> IS $x - \bar{x}$.

x	$x-\bar{x}$	$(x-\bar{x})^2$
8	-7	49
12	-3	9
15	0	0
20	5	25
21	6	36
14	-1	1

ADD UP $(x-\bar{x})^2$, DIVIDE BY $n-1$. $\Rightarrow \frac{120}{5} = 24$

Def: GIVEN DATA $x_1, x_2, \ldots, x_n$ WITH MEAN $\bar{x}$,

VARIANCE $s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$

$$= \frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + \ldots + (x_n-\bar{x})^2}{n-1}$$

Def:

STANDARD DEVIATION

$$S = \sqrt{s^2} \quad \text{SQ. RT. OF VARIANCE}$$

$$= \sqrt{\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + \dots + (x_n-\bar{x})^2}{n-1}}$$

IN PREVIOUS EXAMPLE: 8, 12, 15, 20, 21, 14

$\bar{x} = 15$, $s^2 = 24$, $s = \sqrt{24} \approx 4.899$

ex. DATA: .9, .8, 1.1, 2.8, 1.4

FIND MEAN $\bar{x}$ & STANDARD DEVIATION S.

$$\bar{x} = \frac{.9 + .8 + \dots + 1.4}{5} = \frac{7}{5} = 1.4$$

$$\boxed{\begin{array}{l} \bar{x} = 1.4 \\ S = .8155 \end{array}}$$

x	$x - \bar{x}$	$(x-\bar{x})^2$
.9	-.5	.25
.8	-.6	.36
1.1	-.3	.09
2.8	1.4	1.96
1.4	0	0

NOTE:

$(\text{ANY } \#)^2 \geq 0$

ADD THESE UP,

DIV. BY $n-1 = 4$

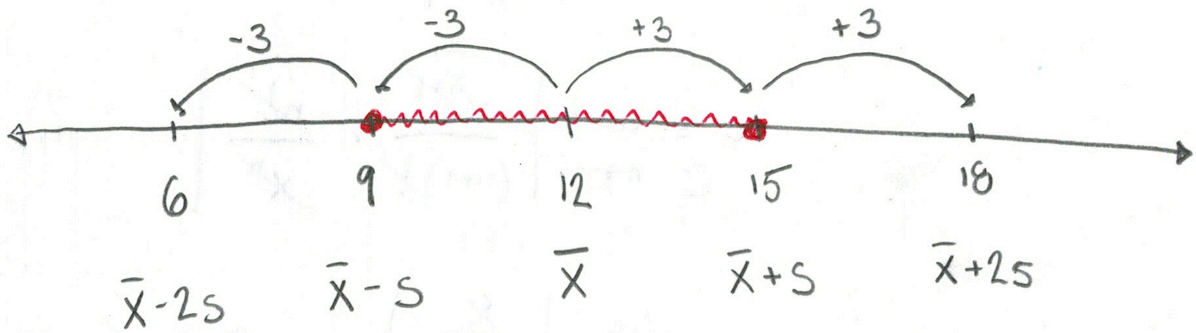
$$s^2 = \frac{2.66}{4} = .665 \rightarrow S = \sqrt{s^2} = \sqrt{.665}$$

$$\approx .8155$$

COUNTING UP AND DOWN BY s,

STARTING AT $\bar{x}$.

$\bar{x} = 12$, $s = 3$



Q: WHAT INTEGERS LIE WITHIN $\underbrace{1s \text{ OF } \bar{x}}$?

($\bar{x}=12$, $s=3$)

WITHIN 3 OF 12?

(INCLUDE ENDPOINTS)

$\boxed{9, 10, 11, 12, 13, 14, 15}$

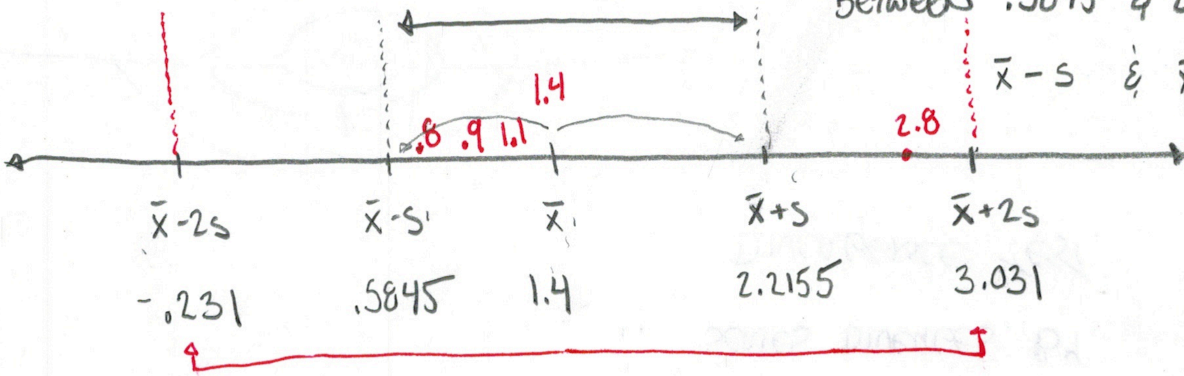
ex. DATA .9 .8 1.1 2.8 1.4

MEAN $\bar{x} = 1.4$, STAND. DEV. $s = .8155$

Q: WHAT PERCENT OF DATA LIES $\underbrace{\text{WITHIN } 1s \text{ OF } \bar{x}}$?

BETWEEN .5845 & 2.2155

$\bar{x} - s$ & $\bar{x} + s$



START AT $\bar{x}$,

TAKE STEPS OF SIZE s (UP & DOWN)

ANSWER: 4 OUT OF 5 $= \frac{4}{5} = .8 =$ 80%

Q: WHAT PERCENT OF DATA LIES WITHIN

$2s$ OF $\bar{x}$?

A: 100% (ALL DATA VALUES)

ex. DATA: 4.2, 5.1, 3.8, 7.2, 1.5, 2.2.

(a) FIND % OF DATA THAT LIE WITHIN 1 STAND DEV. OF MEAN.

(b) 〃 〃 〃 〃 〃 〃 2 〃 〃 〃.

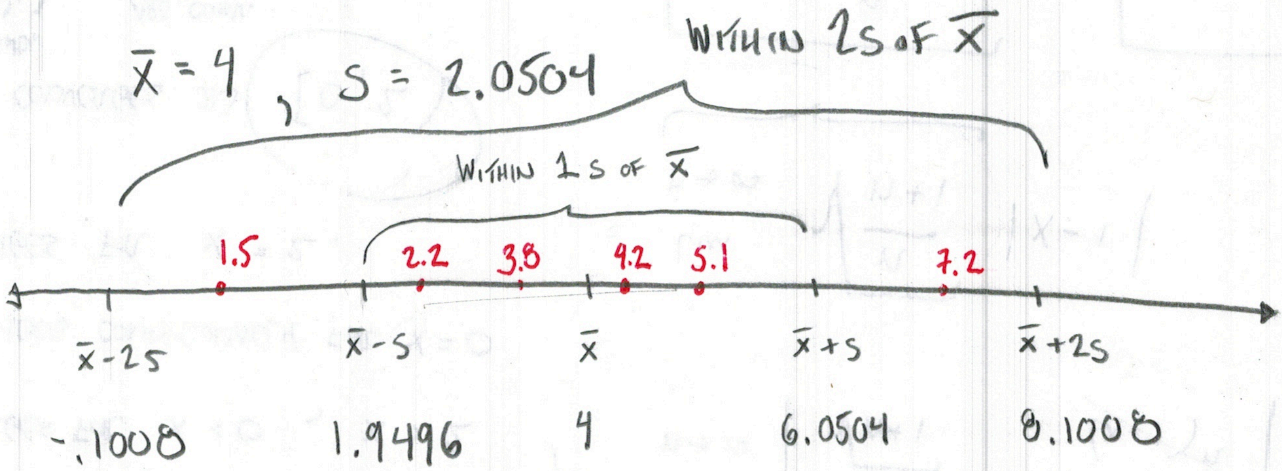
$$\text{MEAN } \bar{x} = \frac{\text{SUM OF } x}{n} = \frac{4.2 + \dots + 2.2}{6} = \frac{24}{6} = 4$$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
4.2	.2	.04
5.1	1.1	1.21
3.8	-.2	.04
7.2	3.2	10.24
1.5	-2.5	6.25
2.2	-1.8	3.24

$$s^2 = \frac{\text{SUM}}{n-1} = \frac{21.02}{6-1} = 4.204$$

$$s = \sqrt{s^2} = \sqrt{4.204} \approx 2.0504$$

$\bar{x} = 4$, $s = 2.0504$



(a) $\frac{4}{6} = 66.7\%$

(b) ALL $= 100\%$

CHEVYCHEV'S THEOREM

GIVEN ANY DATA SET,

THE PROPORTION OF VALUES WITHIN $\underline{k}$

STANDARD DEVIATIONS OF THE MEAN IS

AT LEAST $\left(1 - \frac{1}{k^2}\right)$.

e.g. % OF DATA VALUES THAT LIE WITHIN

2 STANDARD DEV. OF MEAN IS AT LEAST

$$1 - \frac{1}{2^2} = 1 - \frac{1}{4} = \frac{3}{4} \text{ or } 75\%$$

e.g. % OF DATA THAT LIE WITHIN $3s$ OF $\overline{x}$?

$k = 3$ ANSWER $= 1 - \frac{1}{3^2} = 1 - \frac{1}{9} = \frac{8}{9}$

$\approx \underline{88.89\%}$