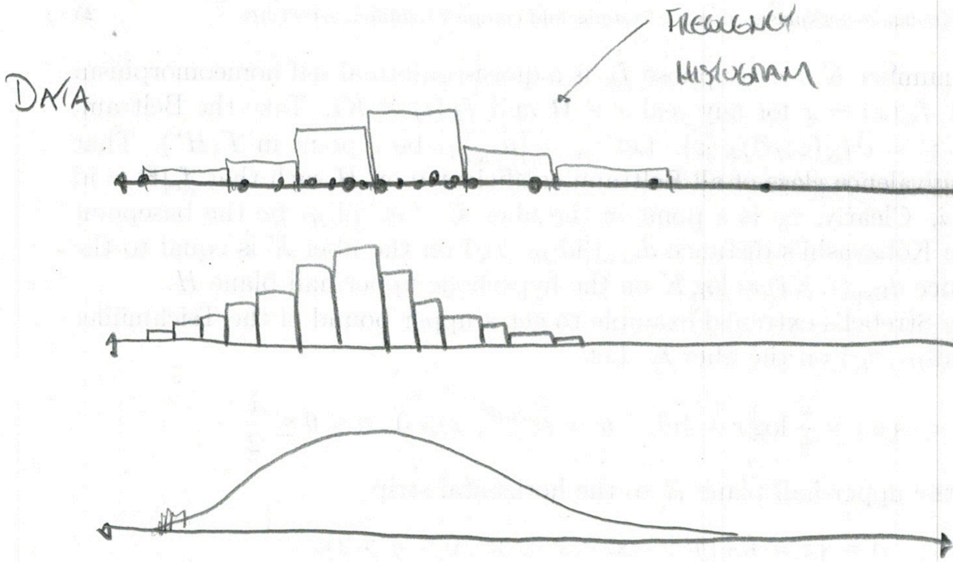
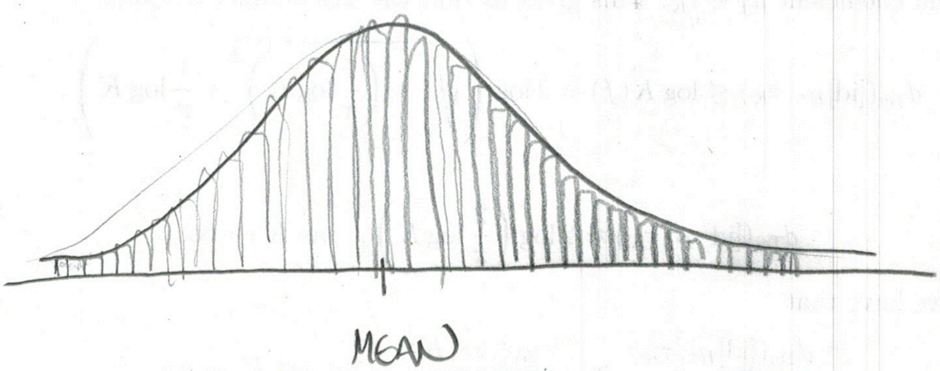
§ 9.3 THE NORMAL DISTRIBUTION



NORMAL DISTRIBUTION

- MOST OF DATA IS CLUSTERED NEAR THE MEAN.
- FARTHER OUT YOU GO ON EITHER SIDE, THESE VALUES ARE MORE AND MORE RARE



SAMPLE MEAN $\bar{X}$

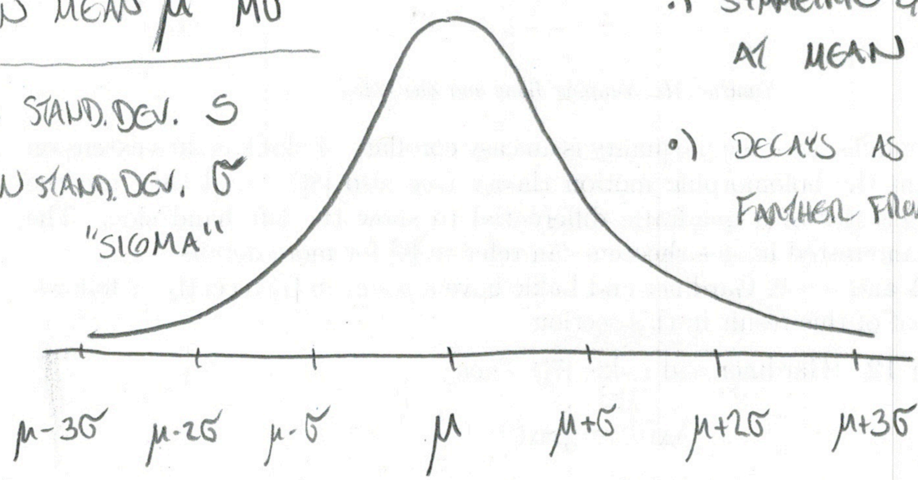
POPULATION MEAN μ "MU"

SAMPLE STAND. DEV. S

POPULATION STAND. DEV. σ "SIGMA"

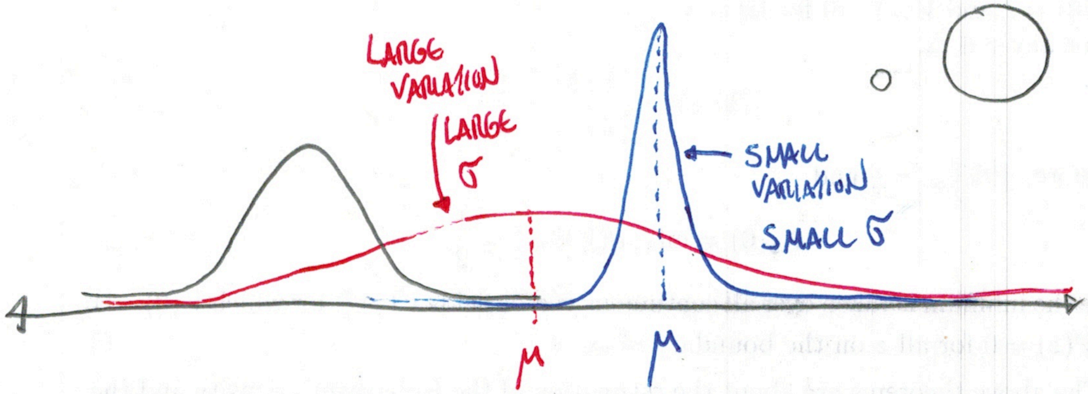
NORMAL DISTRIBUTION

- SYMMETRIC & CENTERED AT MEAN
- DECAYS AS YOU GO FARTHER FROM THE MEAN



ALL NORMAL DISTRIBUTIONS HAVE THE EXACT SAME SHAPE, JUST DIFFERENT μ , σ

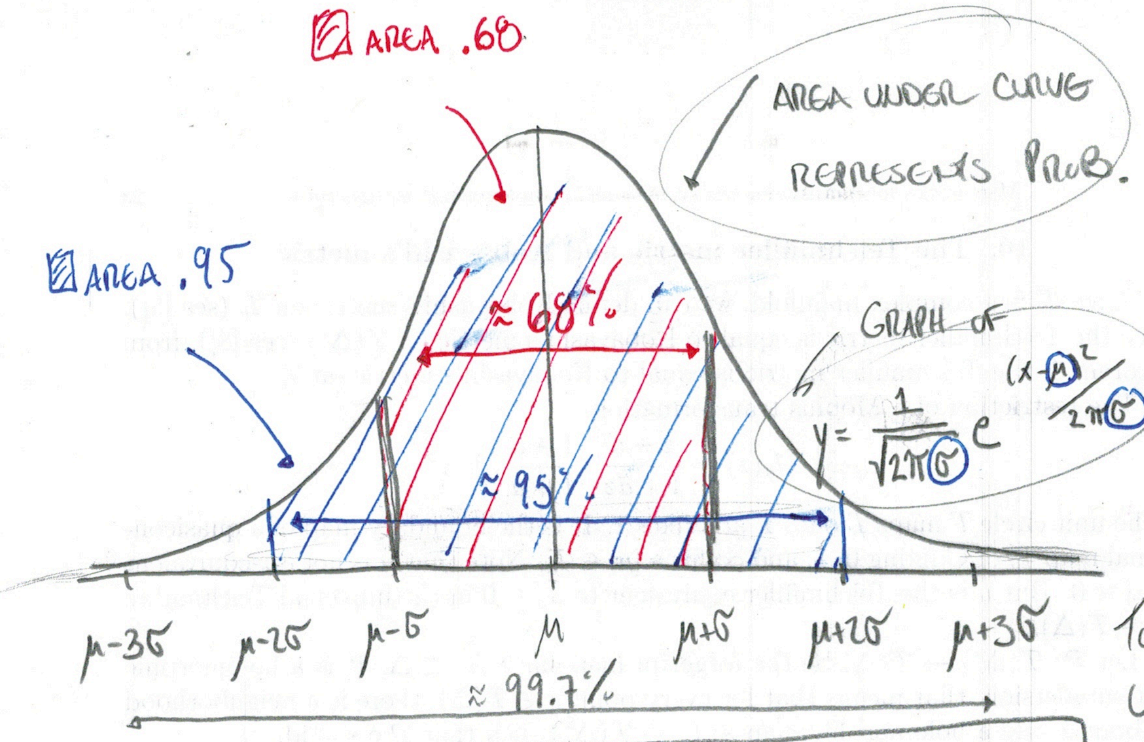
MEAN — STANDARD DEV.



ONLY DIFFERENCE IS CENTER μ

& HOW MUCH THEY'VE BEEN STRETCHED / COMPRESS HORIZONTALLY (& STRETCHED / COMPRESSED VERTICALLY)

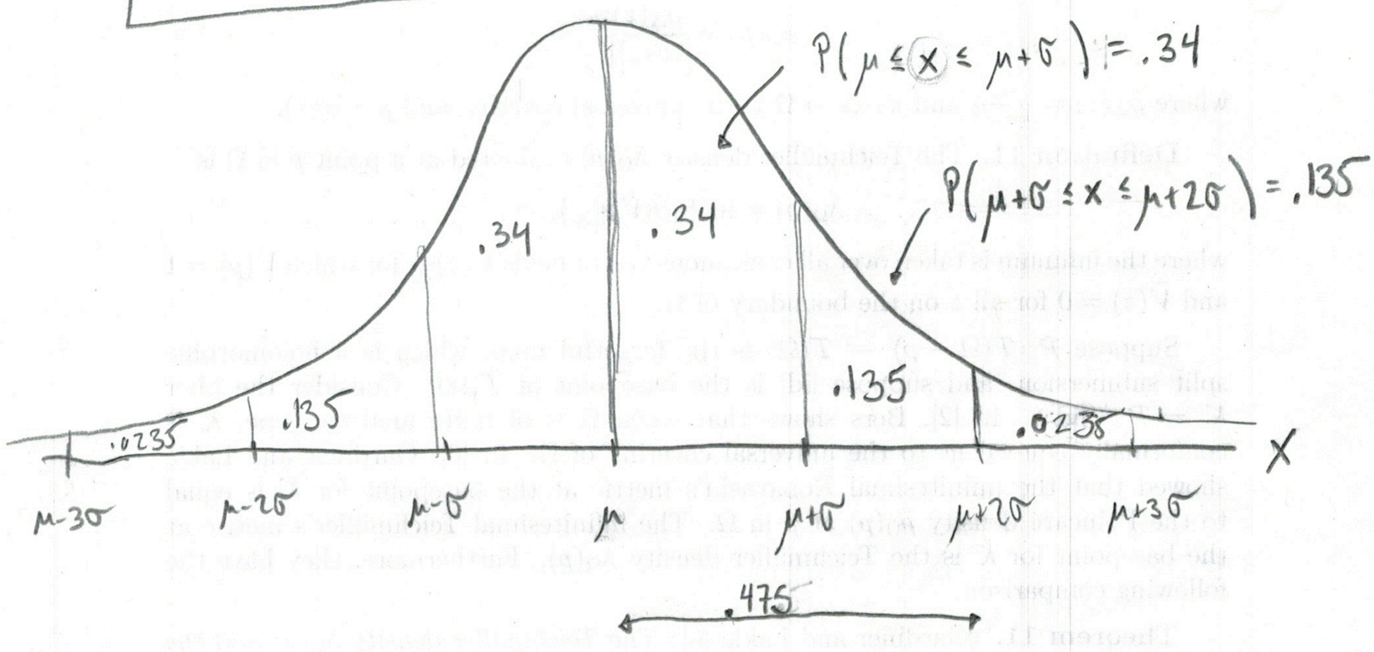
↳ DETERMINED BY σ.



TOTAL AREA UNDER THE CURVE IS 1.

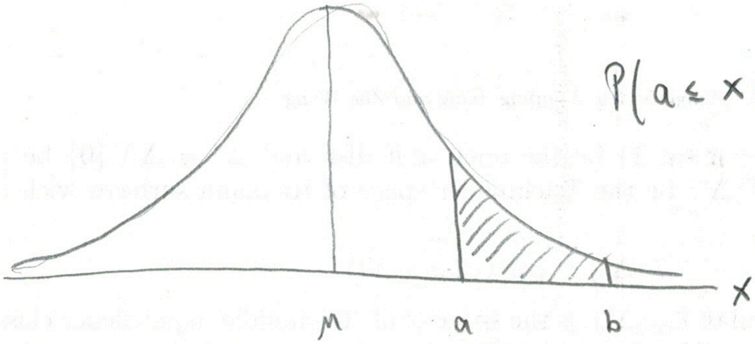
EMPERICAL RULE

≈ 68% OF DATA LIES WITHIN 1 σ OF μ.
≈ 95% OF DATA LIES WITHIN 2 σ OF μ.
≈ 99.7% OF DATA LIES WITHIN 3 σ OF μ.



X HAS NORMAL DISTR.

BELL CURVE

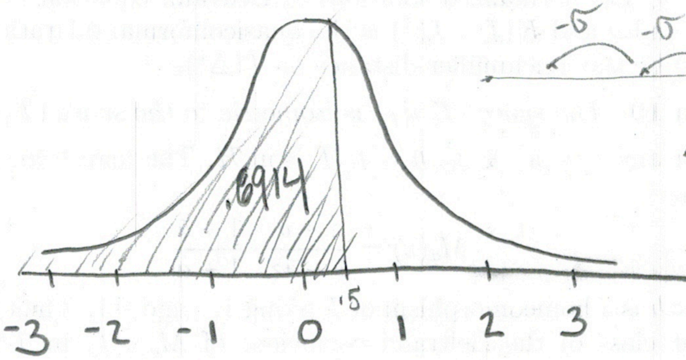


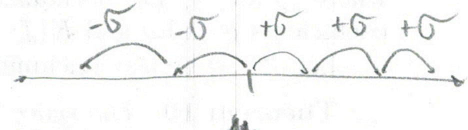
$P(a \le x \le b)$ = AREA UNDER NORMAL CURVE BETWEEN a & b.

STANDARD NORMAL DISTRIBUTION Z

$\mu = 0$

$\sigma = 1$





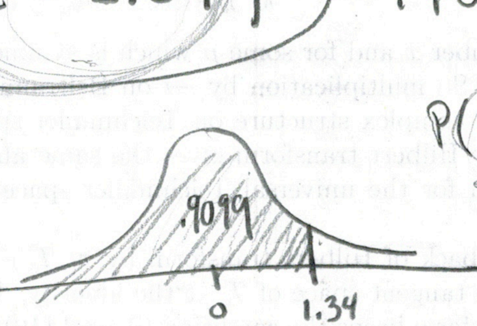
SUPPOSE Z HAS A STANDARD NORMAL DISTRIBUTION

FIND PROBABILITY $P(z \le .5) = .6914$

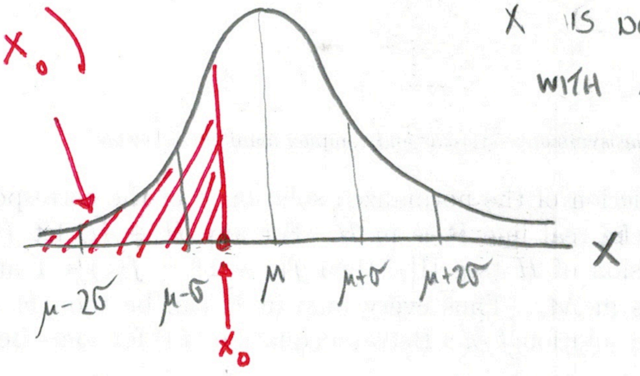
$P(z \le 1.34) = .9099$

$P(z > 1.34) = 1 - .9099$

$= .0901$



AREA = $P(X \le X_0)$

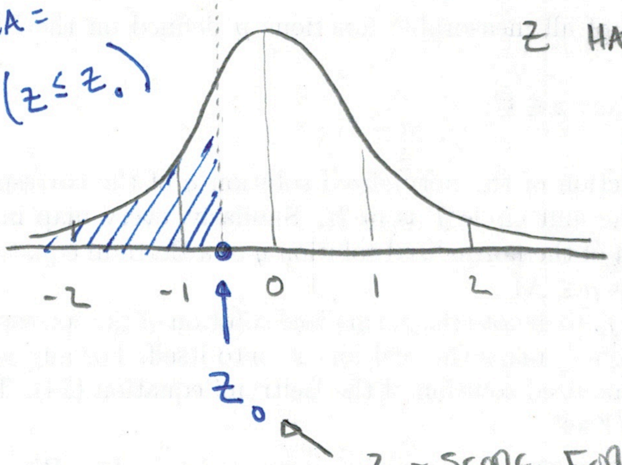


X IS NORMALLY DISTRIBUTED

WITH MEAN μ

STAND DEV σ

AREA = $P(Z \le Z_0)$



Z HAS STANDARD NORMAL DISTR.

MEAN 0

STAND. DEV. 1

Z-SCORE FOR VALUE X_0

Def: GIVEN A VALUE X_0 FOR A NORM. DISTR. RANDOM VARIABLE WITH MEAN μ, STAND. DEV. σ,

THE Z-SCORE FOR X_0 IS THE # OF STAND DEV. ABOVE/BELOW THE MEAN μ IS.

+ −

Z-SCORE: $$Z_0 = \frac{X_0 - \mu}{\sigma}$$ ← DEVIATION FROM MEAN

e.g. SUPPOSE X HAS NORM DISTR. WITH MEAN 82, STAND DEV. 6.

FIND Z-SCORE FOR $X_0 = 87$.

$$Z_0 = \frac{87 - 82}{6} = \frac{5}{6} \approx .83$$

FIND $P(X \ge 87) = 1 - P(X \le 87) = 1 - P(Z \le .83) = \boxed{.2033}$

$1 - .7967$

IN SUMMARY,

SUPPOSE X HAS NORMAL DISTR. WITH MEAN μ & STAND. DEV. σ.

THEN $P(X \le x_0) = P\left(Z \le \dfrac{x_0 - \mu}{\sigma}\right)$

STANDARD NORMAL DISTRIBUTION

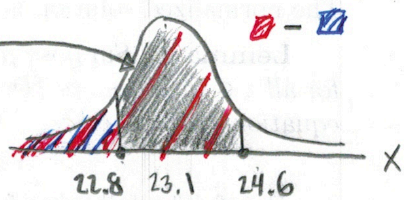
Z-SCORE

e.g. SUPPOSE X HAS NORM. DISTR. WITH $\mu = 23.1$

$\sigma = 2.5$

AREA

FIND $P(\underbrace{22.8 \le X \le 24.6}_{x \le 24.6 \text{ BUT } x \text{ NOT} \le 22.8})$



$= P(X \le 24.6) - P(X \le 22.8)$

$= P\left(Z \le \dfrac{24.6 - 23.1}{2.5}\right) - P\left(Z \le \dfrac{22.8 - 23.1}{2.5}\right)$

$= P(Z \le .6) - P(Z \le -.12)$

$= .7257 - .4522$

$= .2735$