§9.4 NORMAL APPROXIMATION TO BINOMIAL DISTRIBUTIONS

LET X BE A BINOMIAL RANDOM VARIABLE WITH n TRIALS & PROBABILITY OF SUCCESS p (& PROBABILITY OF FAILURE $q = 1-p$).

WE KNOW $P(X = k) = C(n,k)\, p^k q^{n-k}$

BUT WHAT IF WE WANT TO FIND $P(X \le k)$?

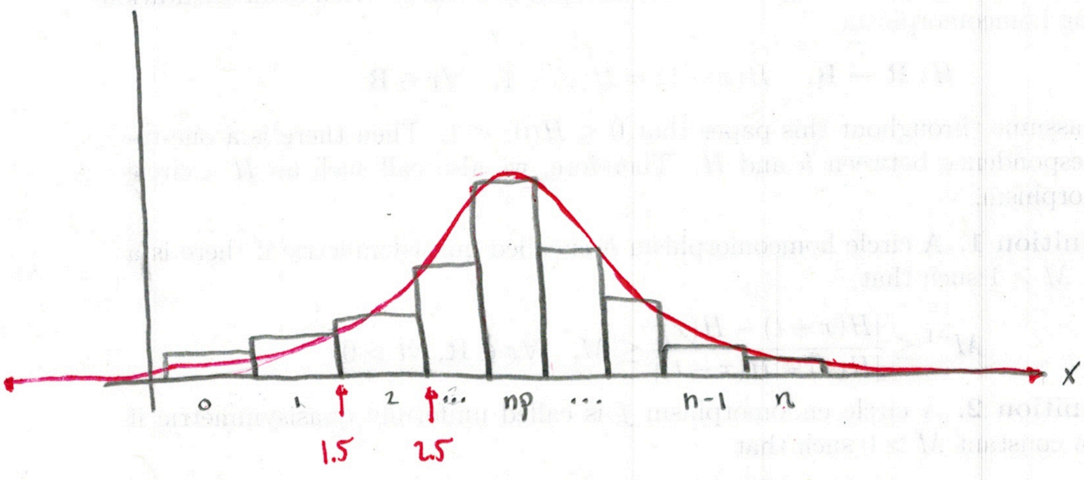
$$P(X \le k) = P(X=0) + P(X=1) + \dots + P(X=k)$$

BETTER WAY? OBSERVE:

X IS BINOMIAL WITH n TRIALS, PROB. SUCCESS p

X HAS MEAN $\mu = np$

STANDARD DEVIATION $\sigma = \sqrt{npq}$



ACTUAL DISTRIBUTION (BINOMIAL)

NORMAL DISTRIBUTION WITH MEAN $\mu = np$, STANDARD DEV $\sigma = \sqrt{npq}$

IN SUMMARY:

IF X IS BINOMIAL WITH n TRIALS & PROB SUCCESS p, FAILURE $q = 1 - p$

* AND IF $np \geq 5$, $nq \geq 5$ *

THEN THE BINOMIAL DISTRIBUTION FOR X IS APPROXIMATELY NORMAL

WITH $\mu = np$, $\sigma = \sqrt{npq}$

e.g. X BINOMIAL

$n = 10$

$p = .5$

i) USE NORMAL DISTRIBUTION TO APPROXIMATE $P(2 \leq X \leq 6)$

ii) $P(2 < X < 6)$

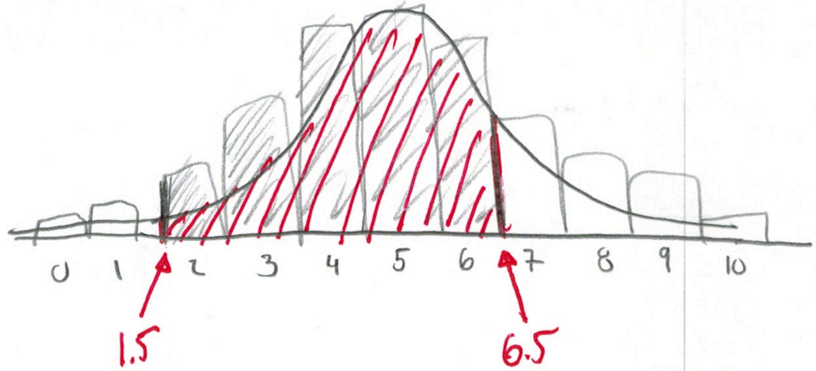
iii) $P(X = 6)$

e.g. ASSUME 20% OF US ADULTS SUBSCRIBE TO AMAZON PRIME.

AND WE RANDOMLY SAMPLE 500.

FIND PROB LESS THAN 80 PEOPLE SUBSCRIBE.

" " MORE THAN 110 PEOPLE SUBSCRIBE.



MEAN : $\mu = np = (10)(.5)$

$= 5$

STAND. DEV : $\sigma = \sqrt{npq}$

$= \sqrt{(10)(.5)(.5)}$

$= \sqrt{2.5} \approx 1.5811$

$P(2 \le X \le 6)$

$P(1.5 \le X_{NORM} \le 6.5)$

$z = \dfrac{x - \mu}{\sigma}$

$$P\left(\frac{1.5-5}{1.5811} \le z \le \frac{6.5-5}{1.5811}\right) = P(-2.21 \le z \le .95)$$

$$= P(z \le .95) - P(z \le -2.21)$$

$$= .8289 - .0136 = \boxed{.8153}$$

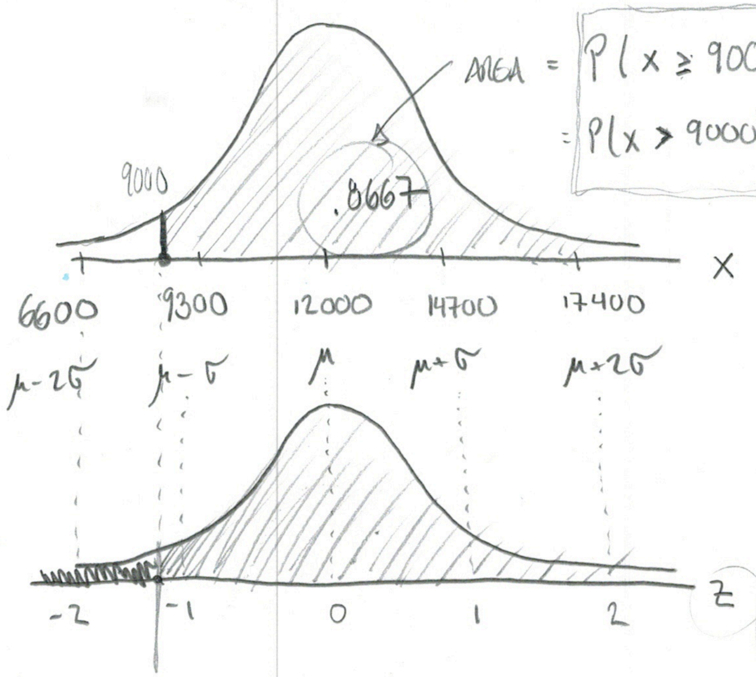
AND HERE ARE THE EXAMPLES FROM

THE BEGINNING OF CLASS

(PEARSON § 9.3 # 17 : 9.3.39)

MEAN $\mu = 12000$

STAN. DEV. $\sigma = 2700$



$$\text{AREA} = \boxed{P(x \geq 9000) = P(x > 9000) + \underbrace{P(x = 9000)}_{0}}$$

STANDARD NORMAL DISTRIBUTION

OF STAN. DEV. ABOVE/BELOW MEAN

$$z = \frac{x - \mu}{\sigma} = \frac{9000 - 12000}{2700}$$

$\boxed{\text{DISTR}}$ → NORMALCDF

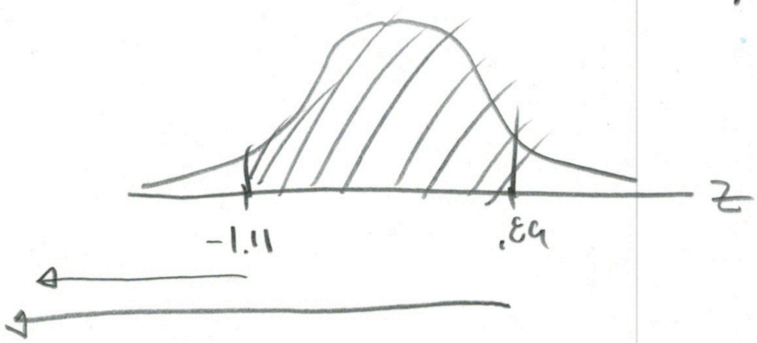
(LOWER , UPPER , μ , σ)

$$P(x > 9000) = P(z > -1.11)$$

$$= 1 - P(z \leq -1.11) = 1 - .1335 = \boxed{.8665}$$

$$P(9{,}000 \leq x \leq 14{,}400)$$

$$= P\left(\frac{9000 - 12000}{2700} \leq z \leq \frac{14400 - 12000}{2700}\right) = P(-1.11 \leq z \leq .89)$$

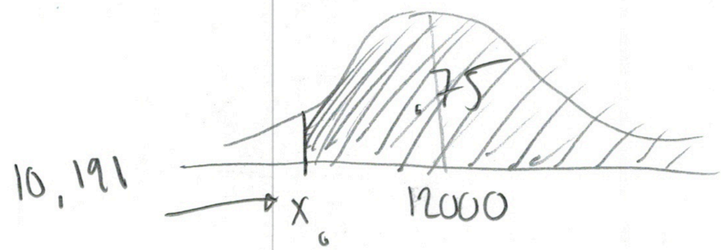


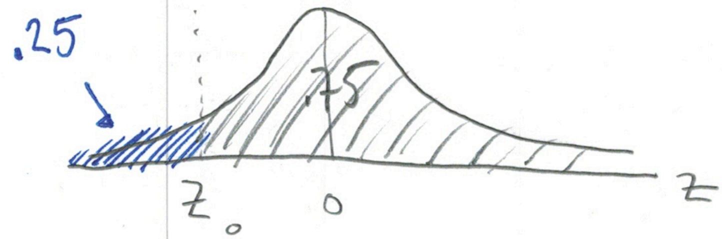
$$= P(z \leq .89) - P(z \leq -1.11)$$

$$= .8133 - .1335$$

$$= .6798$$

FIND X_0 SUCH THAT $P(x \geq x_0) = .75$





$$P(z \leq -.67) = .2514$$

$$P(z \geq -.67) = .7486 \approx .75$$

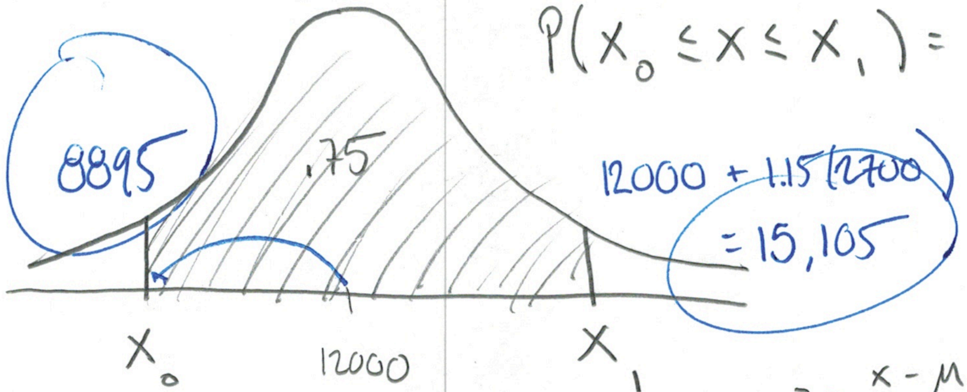
$$z = \frac{x - \mu}{\sigma} \rightarrow -.67 = \frac{x - 12000}{2700}$$

$$(-.67)2700 = x - 12000$$

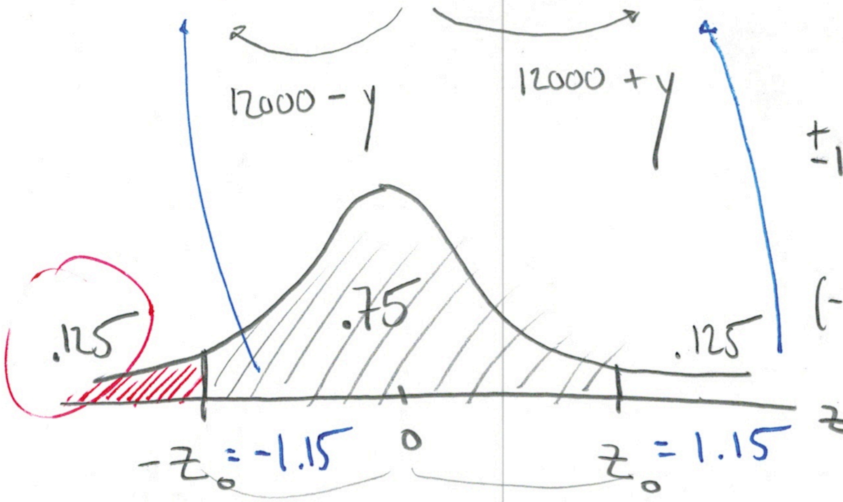
$$12000 - .67 \times 2700 = x$$

$$\underline{10,191 = x}$$

$P(x_0 \le x \le x_1) = .75$



$z = \frac{x - \mu}{\sigma}$



$\pm 1.15 = \frac{x - 12000}{2700}$

$(-1.15)(2700) = x - 1200$

$x = 12000 \pm 1.15(2700)$

$x = 8895$

$P(z \le -z_0) = .125$

$\uparrow$

$-z_0 = -1.15$

$z_0 = 1.15$