CHAPTER 9 REVIEW

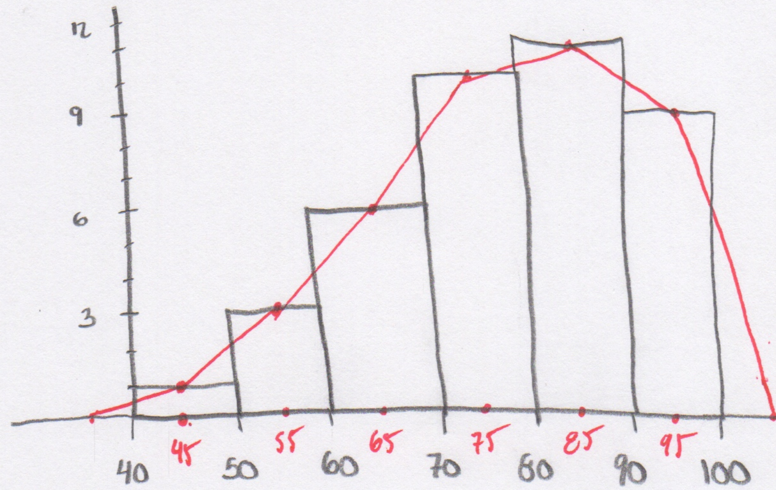
§ 9.1 ·) VISUALIZING THE DISTRIBUTION OF DATA (NUMBERS) WITH A FREQUENCY HISTOGRAM

DATA :

43	51	54	59	60	61	61	64	64	68
70	70	73	73	73	73	77	77	79	79
80	80	80	80	80	83	88	88	88	88
89	91	91	92	95	95	95	96	96	98

RANGE: 98 − 43 = 55

	CLASS	FREQUENCY
45	$40 \leq x < 50$	1
55	$50 \leq x < 60$	3
65	$60 \leq x < 70$	6
75	$70 \leq x < 80$	10
85	$80 \leq x < 90$	11
95	$90 \leq x < 100$	9



$$\text{MEDIAN} = \frac{75+85}{2} = 80$$

NOTE THAT ENDPOINTS ARE INCLUDED IN ONLY ONE CLASS.

FREQUENCY POLYGON

IF ONLY THE TABLE OF HISTOGRAM IS PROVIDED, ASSUME ALL DATA VALUES BELONGING TO EACH CLASS ARE THE MIDPOINT OF THAT CLASS:

$$\frac{\text{LEFT ENDPOINT} + \text{RIGHT ENDPOINT}}{2}$$

- SUMMARIZING THE DISTRIBUTION OF DATA WITH A MEASURE OF CENTER:

POPULATION

GREEK LETTERS μ

(1) MEAN: $\bar{x} = \frac{\sum x}{n}$ — X ARE THE DATA VALUES

SUM OF ALL VALUES ÷ # OF VALUES

(2) MEDIAN: SORT VALUES LEAST TO GREATEST

VALUE IN MIDDLE POSITION / MEAN OF 2 VALUES THAT SHARE THE MIDDLE.

2 $\boxed{5}$ $\boxed{6}$ 9

5.5

(3) MODE:

VALUE WITH GREATEST FREQUENCY.

§9.2 HOW TO MEASURE VARIATION OF A DISTRIBUTION OF VALUES

- RANGE = MAX − MIN

- VARIANCE $s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$

- STANDARD DEVIATION $s = \sqrt{s^2}$

e.g. 8, 9, 15, 16, 22

EVERY VALUE X HAS A CORRESPOND DEVIATION FROM THE MEAN, $\bar{X}$.

$x - \bar{x}$ (TAKE ON SCALE)

8, 9, 15, 16, 22

FIND MEAN $\bar{x}$ &

STAND. DEV. σ

MEAN:

$$\bar{x} = \frac{8 + 9 + \dots + 22}{5} = \frac{70}{5} = 14$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
8	-6	36
9	-5	25
15	1	1
16	2	4
22	8	+ 64

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

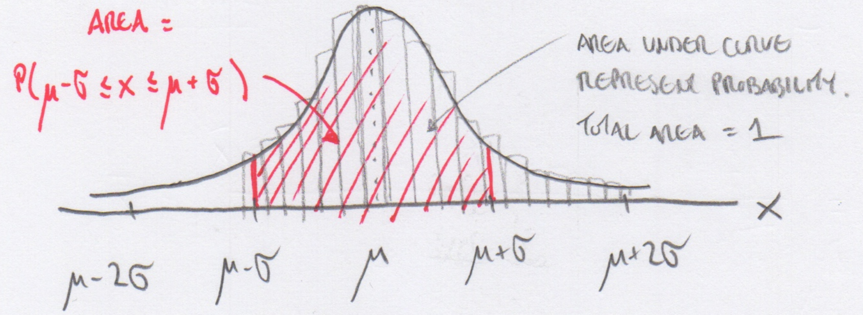
$$= \frac{130}{5 - 1} = \frac{130}{4} = 32.5$$

VARIANCE $s^2 = 32.5$

STAND. DEV. $s = \sqrt{s^2} = \sqrt{32.5} = 5.7009$

§ 9.3

GIVEN A RANDOM VARIABLE X WITH A NORMAL DISTRIBUTION



MEAN μ

STANDARD DEV. σ

EVERY VALUE X HAS A CORRESPONDING Z-SCORE

$$Z = \frac{X - \mu}{\sigma}$$

THIS IS CALLED STANDARDIZING THE RANDOM VARIABLE X.

Z IS CALLED THE STANDARD NORMAL RANDOM VARIABLE.

Z HAS MEAN 0 STAN. DEV. 1

$$P(X \le x_0) = P(Z \le z_0)$$

WHERE $z_0 = \dfrac{x_0 - \mu}{\sigma}$

e.g. SUPPOSE X IS A NORMALLY DISTRIBUTED RANDOM VARIABLE WITH MEAN $\mu = 65$ & STANDARD DEVIATION $\sigma = 8$.

(a) FIND $P(X \le 60)$.

(b) FIND $P(X \ge 60)$.

(c) FIND $P(60 \le X \le 75)$.

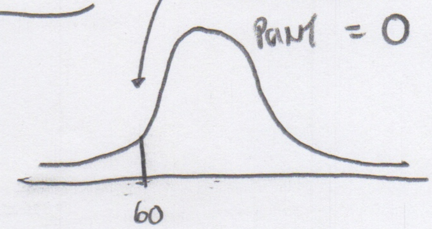
(a) $P(x \leq 60) = P\left(z \leq \frac{60-65}{8}\right)$

$= P(z \leq -.6250) \approx P(z \leq -.63)$

$= \boxed{.2643}$

(b) $P(x \geq 60) = \underbrace{P(x > 60)}_{\substack{\uparrow \\ \text{COMPLEMENT OF } x \leq 60}} + \underbrace{P(x = 60)}_{0}$

AREA UNDER CURVE ABOVE POINT = 0



$= 1 - P(x \leq 60) = 1 - .2643$

$= \boxed{.7357}$

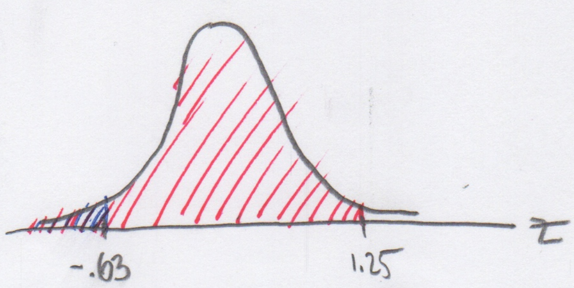
(c) $P(\underset{(1)}{60} \leq x \leq \underset{(2)}{75}) = P(-.63 \leq z \leq 1.25)$

(1) $z = \frac{60-65}{8} = -.63$

$= P(z \leq 1.25$ BUT z IS <u>NOT</u> $\leq -.63)$

(2) $z = \frac{75-65}{8} = 1.25$

$= P(z \leq 1.25) - P(z \leq -.63)$



$= .8944 - .2643$

$= \boxed{.6301}$

§9.4 A BINOMIAL RANDOM VARIABLE X

X = # SUCCESSES IN n TRIALS,
EACH WITH PROBABILITY OF SUCCESS p,
PROBABILITY OF FAILURE $q = 1 - p$.

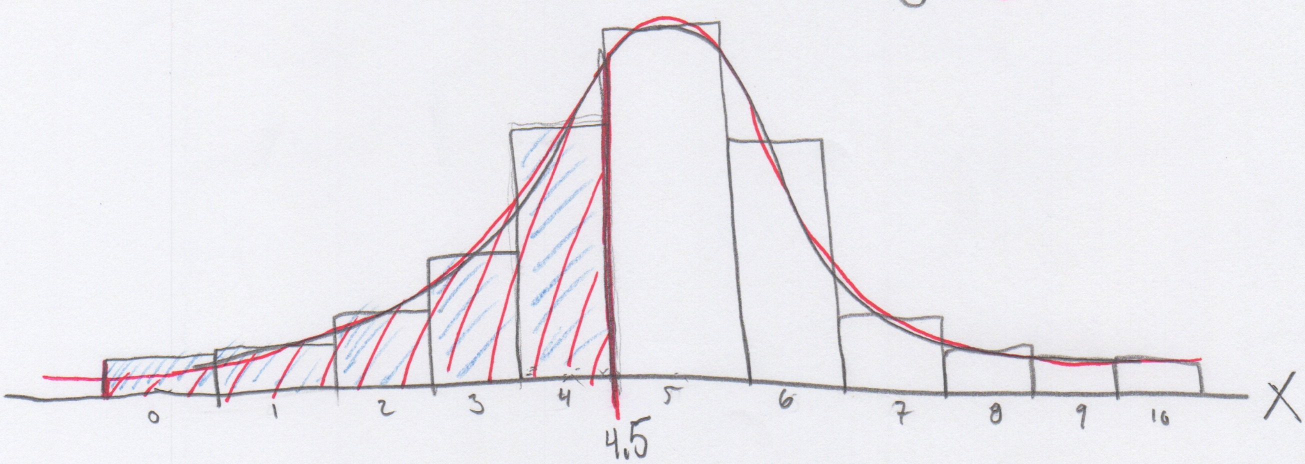
* ASSUME $np \geq 5$, $nq \geq 5$ *

IS APPROXIMATELY NORMALLY DISTRIBUTED WITH

MEAN $\mu = np$

STANDARD DEVIATION $\sigma = \sqrt{npq}$

MEAN & S.D. FOR BIN. RAN. VAR.



SUPPOSE X IS BINOMIAL WITH $n = 10$, $p = .5$

THEN $P(x \leq 4) \approx P\left(z \leq \frac{4.5 - \mu}{\sigma}\right) = P\left(z \leq \frac{4.5 - 5}{1.5811}\right)$*

$P(x=0) + P(x=1) + \ldots + P(x=4)$

$C(10,0)(.5)^0(.5)^{10} + \ldots$

$\mu = np = (10)(.5) = 5$

$\sigma = \sqrt{npq} = \sqrt{(10)(.5)(.5)}$

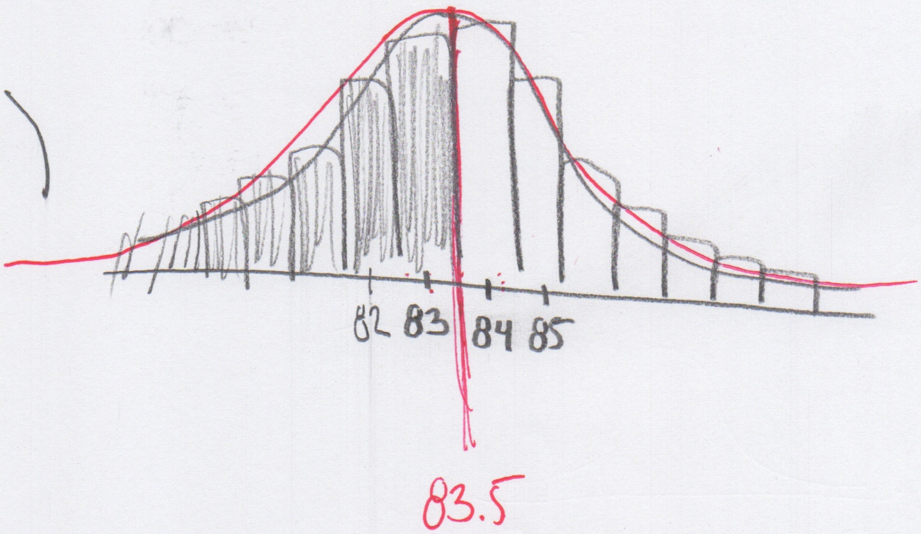
$= \sqrt{2.5} = 1.5811$

* $P(z \leq -.79) = \boxed{.2148}$

X IS BINOMIAL WITH $\mu = 80$ $\quad n = \ldots$

$\sigma = 6$ $\quad p = \ldots$

$P(X < 84)$



$$P(78 < X < 86) \approx P\left(\frac{78.5-80}{6} \leq z \leq \frac{85.5-80}{6}\right)$$