Name: * ANSWER KEY * 2/13/2020

Math 1100-L01: Finite Mathematics Exam 1

Please put away all papers, phones, smart watches, headphones, and electronic devices *except* a calculator. Answer all of the following questions. Show enough work that it is clear how you arrived at your answer. Put a box/circle around your final answer to each question. Good luck!

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set, and let $A = \{2, 3, 5, 7\}$ and $B = \{1, 3, 5, 7, 9\}$ be subsets of U.

   (a) (2 points) True or false: $\emptyset \subseteq U$ TRUE

   (b) (2 points) True or false: $A \in U$ FALSE

   (c) (2 points) True of false: $B \subseteq B$ TRUE

   (d) (4 points) Find $A \cup B$.

   $\{1, 2, 3, 5, 7, 9\}$

   THESE ARE THE ELEMENTS IN EITHER A OR B.

   (e) (4 points) Find $A \cap B'$.

   $\{2\}$

   NOTE: $(A \cap B')' = A' \cup B$

   $(A' \cup B)' = A \cap B'$

   THESE ARE THE ELEMENTS IN A AND NOT IN B.

   (f) (4 points) Find $A' \cup B$.

   $\{1, 3, 4, 5, 6, 7, 8, 9\}$

   THESE ARE THE ELEMENTS THAT ARE EITHER NOT IN A OR IN B.

   (g) (6 points) List all possible subsets of $C = \{a, b, c\}$.

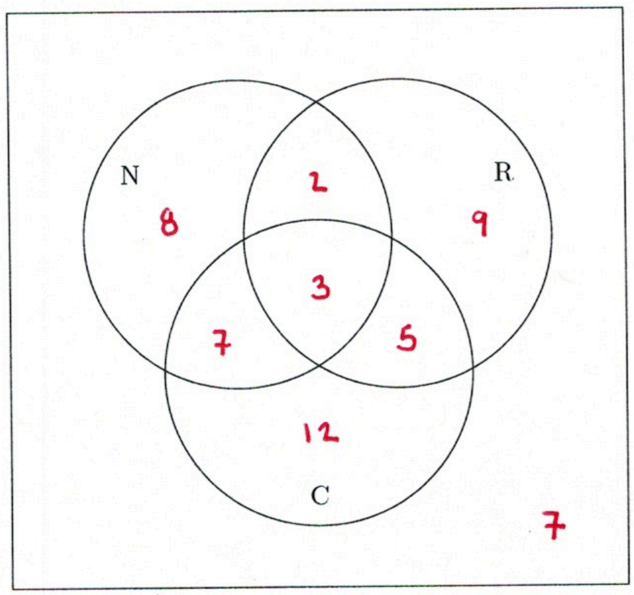
   $\emptyset$ $\{a\}$ $\{a, b\}$ $\{a, b, c\}$

   $\{b\}$ $\{a, c\}$

   $\{c\}$ $\{b, c\}$

2. A telephone survey of TV viewers asked participants whether or not they watch news shows (N), reality shows (R), and/or comedies (C). The results are listed below.

- 20 watch news shows
- 19 watch reality shows
- 27 watch comedies
- 19 watch comedies but not reality shows
- 15 watch news shows but not reality shows
- 10 watch both news shows and comedies
- 3 watch all three
- 7 watch none of these



(a) (6 points) How many TV viewers participated in this survey?

$n(U) = 53$

(b) (6 points) How many TV viewers watch only comedies?

$n(C \cap N' \cap R') = 12$

(c) (6 points) How many TV viewers do not watch comedies?

$n(C') = 26$

3. An experiment consists of randomly selecting 2 marbles (one at a time, without replacement) from a jar that initially contains 8 red marbles, 7 white marbles, and 6 blue marbles.

(a) (6 points) What is the probability that the first marble is red?

8 RED MARBLES

$8 + 7 + 6 = 21$ TOTAL MARBLES

ALL EQUALLY LIKELY

$$\boxed{\frac{8}{21}}$$

(b) (6 points) What is the probability that the first marble is not red?

$$P(\text{NOT RED}) = 1 - P(\text{RED}) = 1 - \frac{8}{21} = \boxed{\frac{13}{21}}$$

(c) (6 points) What is the probability that the second marble is not red, given that the first marble is red?

NOW A RED MARBLE HAS BEEN REMOVED.

SO THERE ARE 7 RED, 7 WHITE, 6 BLUE MARBLES.

$7 + 6 = 13$ MARBLES NOT RED

$7 + 7 + 6 = 20$ MARBLES TOTAL

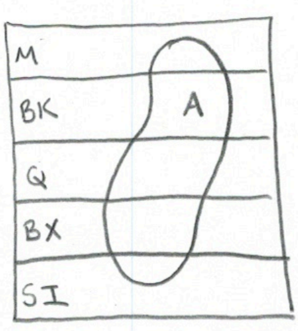
$$\boxed{\frac{13}{20}}$$

4. (6 points) The following table gives the proportion of New York City residents that live in each of the five boroughs, as well as the proportion of residnts in each borough that were born in a foreign country.

Borough	Proportion of NYC residents	Proportion born in a foreign country
Manhattan	.19	.29
Brooklyn	.31	.38
Queens	.27	.49
Bronx	.17	.32
Staten Island	.06	.21

Find the probability that a randomly selected New York City resident was born in a foreign country.

(LAW OF TOTAL PROBABILITY)



$$P(A) = P(A \cap M) + P(A \cap BK) + P(A \cap Q) + P(A \cap BX) + P(A \cap SI)$$

$$= P(M)P(A|M) + P(BK)P(A|BK) + P(Q)P(A|Q)$$

$$+ P(BX)P(A|BX) + P(SI)P(A|SI)$$

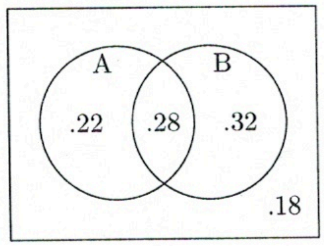
$$= (.19)(.29) + (.31)(.38) + (.27)(.49) + (.17)(.32)$$

$$+ (.06)(.21)$$

$$= \boxed{.3722}$$

5. An experiment can result in events A, B, both A and B, or neither with the following probabilities. (Note: the chart and the Venn diagram are equivalent.)

	A	A'
B	.28	.32
B'	.22	.18



(a) (6 points) Find $P(A)$.

$P(A) = P(A \cap B) + P(A \cap B') = .28 + .22 = \boxed{.5}$

(b) (6 points) Find $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.28}{.28 + .32} = \frac{.28}{.6} \approx \boxed{.4667}$$

(c) (5 points) Are A and B independent? Why or why not?

NO, BECAUSE $P(A) \neq P(A|B)$

$(.5 \neq .4667)$

(d) (5 points) Are A and B mutually exclusive (i.e. disjoint)? Why or why not?

NO, BECAUSE $P(A \cap B) \neq 0$

$(.28 \neq 0)$

6. Suppose that there is a test for a certain disease such that the probability that a person who has the disease tests positive is .999, and the probability that a person who does not have the disease tests positive is .015. Furthermore, suppose that the disease is rare, and the probability that a randomly selected person has the disease is only .001.

(a) (6 points) What is the probability that a randomly selected person tests positive for the disease?

$$P(T) = P(T \cap D) + P(T \cap D')$$

$$= P(D)P(T|D) + P(D')P(T|D') \qquad \text{(LAW OF TOTAL PROBABILITY)}$$

$$= (.001)(.999) + (.999)(.015)$$

$$= .000999 + .014985$$

$$= \boxed{.015984} \approx \boxed{.0160}$$

(b) (6 points) What is the probability that a randomly selected person has the disease, given that they have tested positive for the disease?

$$\text{(BAYE'S THM)} \qquad P(D|T) = \frac{P(D)P(T|D)}{P(T)}$$

$$= \frac{(.001)(.999)}{(.0160)} \qquad\qquad \frac{(.001)(.999)}{(.015984)}$$

$$\approx \boxed{.0624} \qquad\qquad \boxed{.0625}$$