# MODULE #2: §5.3 LINEAR PROGRAMMING GEOMETRIC APPROACH

ex 1. $\underbrace{\text{MAXIMIZE \& MINIMIZE}}_{\text{"OPTIMIZE"}}$ $z = 10x + 30y$

SUBJECT TO CONSTRAINTS:

$$2x + y \geq 16$$
$$x + y \geq 12$$
$$x + 2y \geq 14$$
$$x, y \geq 0 \quad \left( \begin{matrix} x \geq 0 \\ y \geq 0 \end{matrix} \right)$$

$$10x + 30y = z$$

BY SETTING $z$ = DIFFERENT CONSTANTS

WE CREATE "LEVEL CURVES"

↳ CURVES SUCH THAT EVERY POINT ON THE CURVE CORRESPONDS TO AN $x$ & $y$ VALUE THAT CAUSE $z$ TO EQUAL THAT CONSTANT VALUE

$$10x + 30y = c$$
$$y = -\frac{1}{3}x + \frac{c}{30}$$

THIS GENERATES A COLLECTION OF LINES WITH SAME SLOPE, DIFFERENT $y$-INTERCEPTS.

$z$ (or $c$) = 140 IS THE MIN.

SMALLEST VALUE OF $z$ (or $c$) SUCH THAT THE LINE $10x + 30y = c$ INTERSECTS THE SOLUTION REGION.

NO MAXIMUM. THE SOLUTION REGION IS UNBOUNDED.

THM:

**THEOREM 2 Existence of Optimal Solutions**

(A) If the feasible region for a linear programming problem is bounded, then both the maximum value and the minimum value of the objective function always exist.

(B) If the feasible region is unbounded and the coefficients of the objective function are positive, then the minimum value of the objective function exists but the maximum value does not.

(C) If the feasible region is empty (that is, there are no points that satisfy all the constraints), then both the maximum value and the minimum value of the objective function do not exist.

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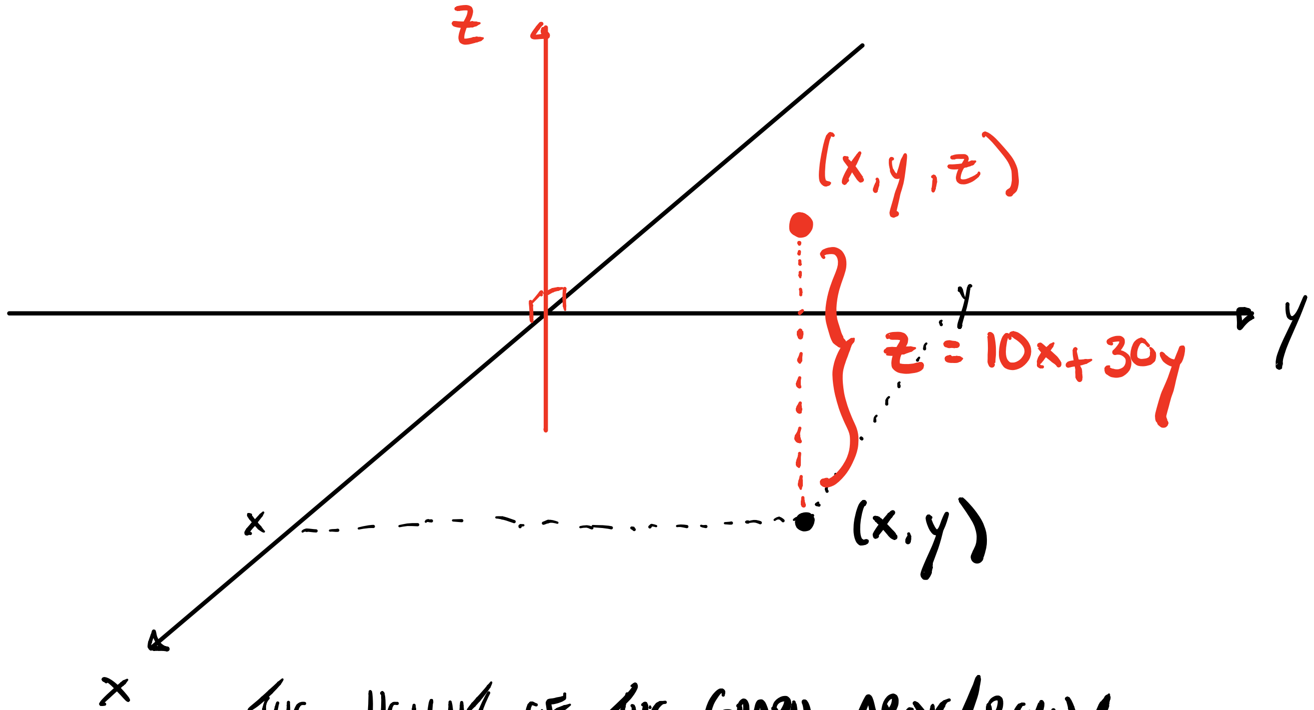
Suppose $z = 7x + 4y$

We can create a graph for this equation. (3 var's)

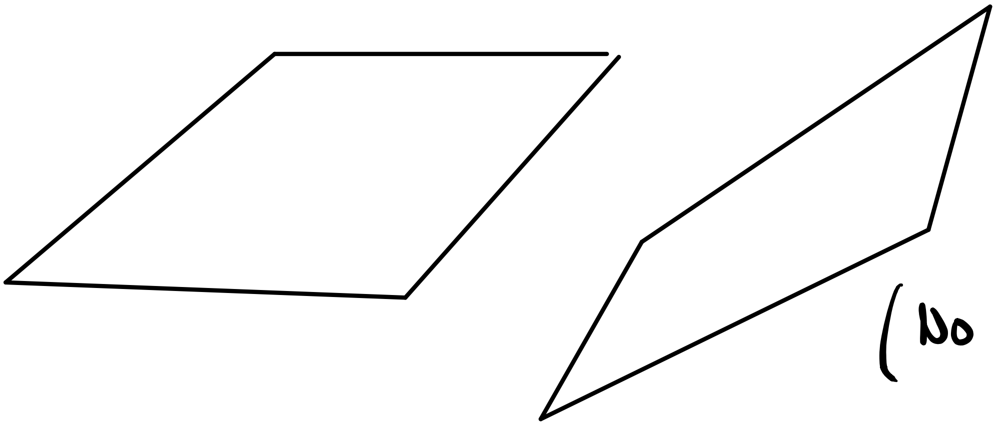



The height of the graph above/below the point $(x, y)$ is the value of $z$ for those values of $x$ & $y$.

The graphs of linear equations in $x, y, z$.

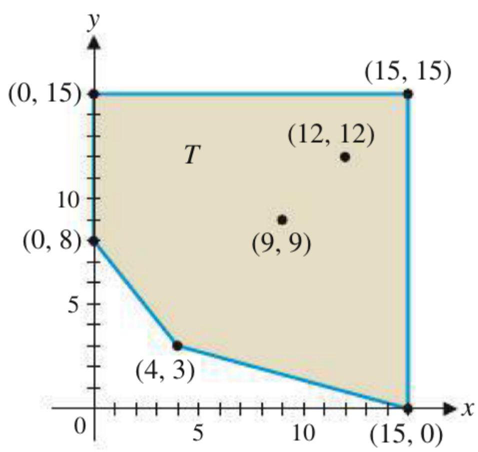
$z = Ax + By$, A, B are constant coefficients

THE GRAPH OF LINEAR EQ'S IN $x, y, z$ ARE PLANES.



(NO CURVATURE)

WHEN MAXIMIZING OR MINIMIZING A LINEAR FUNCTION OF 2 VARIABLES $(x, y)$ OVER A BOUNDED SOLUTION REGION, THE EXTREME VALUES OCCUR ON THE BOUNDARY. WHEN THE BOUNDARY IS COMPOSED OF STRAIGHT LINES, THE EXTREME VALUES OCCUR AT CORNER POINTS OF THE FEASABLE SOLUTION REGION.




**13.** $C = 7x + 4y$ **14.** $C = 7x + 9y$

QUESTION 13: MAX/MIN $C$

IN SOLUTION REGION.

## ex 2.

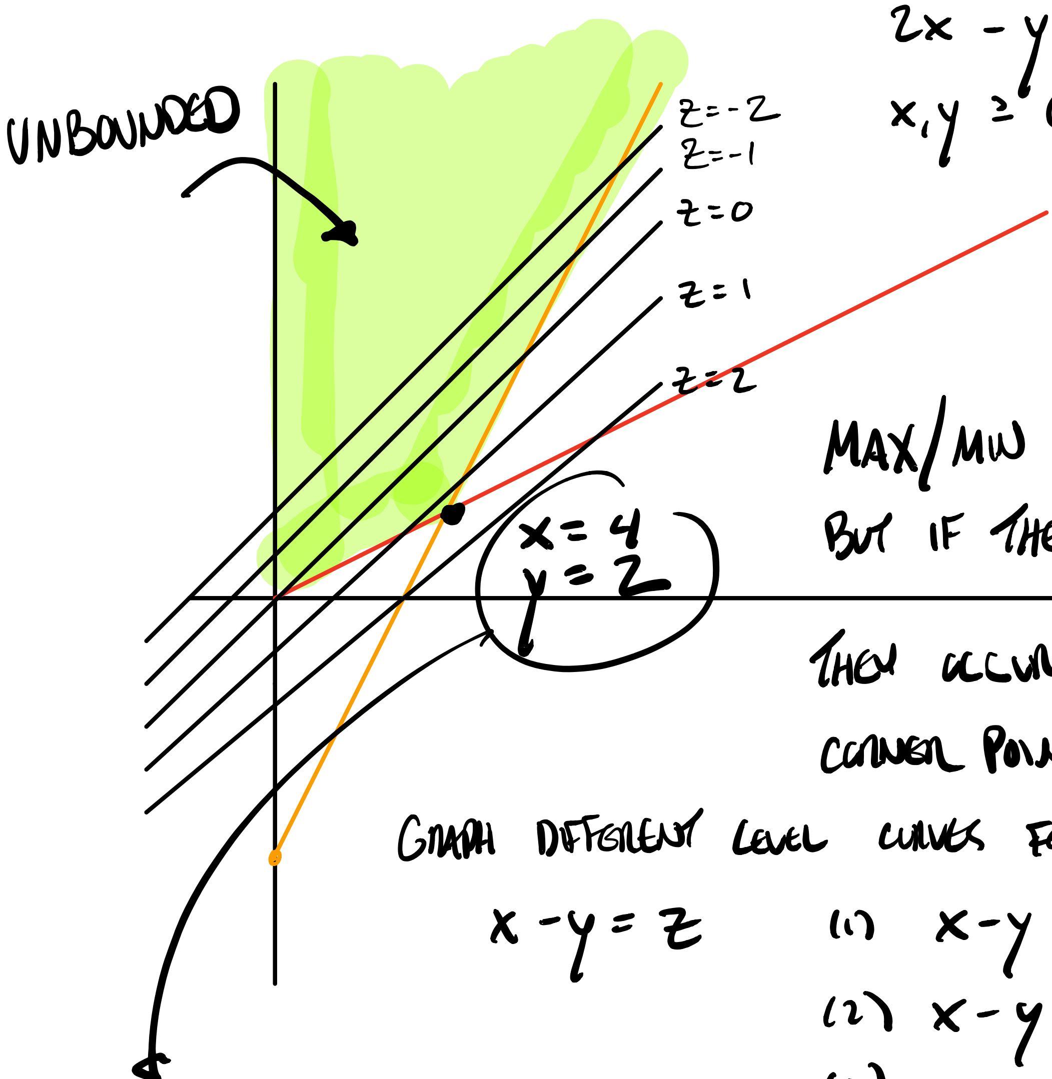
MINIMIZE / MAXIMIZE $z = x - y$

SUBJECT TO CONSTRAINTS:

$x - 2y \le 0$ • $y \ge \frac{1}{2}x$

$2x - y \le 6$ • $y \ge 2x - 6$

$x, y \ge 0$




MAX/MIN NOT GUARANTEED, BUT IF THEY EXIST, THEN OCCUR AT A CORNER POINT.

GRAPH DIFFERENT LEVEL CURVES FOR

$x - y = z$

(1) $x - y = 0$ $\quad y = x$

(2) $x - y = 1$ $\quad y = x - 1$

(3) $x - y = 2$ $\quad y = x - 2$

⋮

(4) $x - y = -1$ $\quad y = x + 1$

(5) $x - y = -2$ $\quad y = x + 2$

MAXIMUM

$z = (4) - (2) = \boxed{2}$

No MINIMUM.

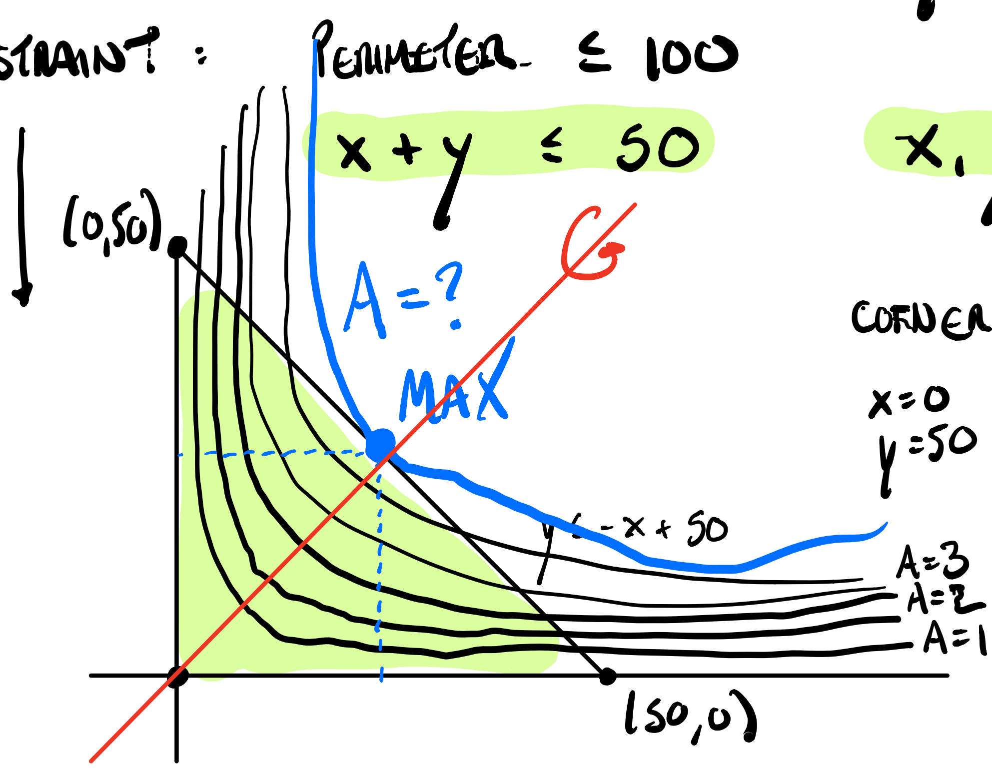
CHALLENGE:

FARMER HAS 100 ft OF FENCING. WANTS TO FENCE IN A RECTANGULAR REGION WITH THE LARGEST POSSIBLE AREA. WHAT DIMENSIONS DO THIS?

MAXIMIZE $A = xy$ (LENGTH: $x$, WIDTH: $y$)

CONSTRAINT: PERIMETER $\leq 100$

$x + y \leq 50$ $\quad$ $x, y \geq 0$




CORNER POINTS

$\left.\begin{matrix} x=0 \\ y=50 \end{matrix}\right\} A = 0$

ALL DIMENSION THAT PRODUCE RECT. WITH SAME AREA:

$A=1 : xy = 1 \to y = \frac{1}{x} \equiv x = \frac{1}{y}$

$A=2 : xy = 2 \to y = \frac{2}{x} \equiv x = \frac{2}{y}$

FAMILY OF CURVES

$A = c : xy = c \to y = \frac{c}{x}$

SWITCHING $x$ & $y$,
WE GET AN EQUIVALENT EQ.

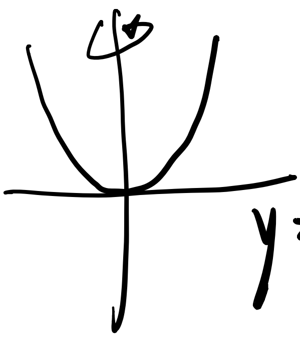
GRAPHICALLY, SWITCHING $x$ & $y$ IS REFLECTION
ACROSS THE DIAGONAL LINE $y=x$.

THESE GRAPHS ARE SYMMETRIC ABOUT LINE $y=x$.

$\therefore \quad x=y=25$ SQUARE

$$A=(25)(25)=\boxed{625.}$$

EQ WITH $x, y$ & REPLACING $x$ WITH $-x$
GIVES THE SAME EQ
THEN $x=0$ IS LINE OF SYMM.



$y=x^2$

$$y=(-x)^2=x^2$$