Module 2: Maximums & Minimums in Linear Programming

ex 1.

27. Minimize and maximize

$z = 10x + 30y$

subject to
$$2x + y \geq 16$$
$$x + y \geq 12$$
$$x + 2y \geq 14$$
$$x, y \geq 0$$

Corner Points	Objective Func
$x=14, y=0$	$10(14) + 30(0) = 140$ MIN
$x=0, y=16$	$10(0) + 30(16) = 480$
$x=4, y=8$	$10(4) + 30(8) = 280$
$x=10, y=2$	$10(10) + 30(2) = 160$

(1) Graph solution region.

- When searching for max/min values of objective functions, it is important that the solution region includes its boundaries.

(2) Consider "Level Curves" for the objective function.

- If we set the obs func. equal to a constant k, we can see all (x,y) values that that cause the obs func to equal k. These (x,y) values all lie on a curve.

(could be a straight line)

$$10x + 30y = k \qquad \text{Eq in } x, y$$

$$y = -\frac{1}{3}x + \frac{k}{30}$$ ← Graph of Eq is straight line.

For every value k we get a different level curve.

- The point where a level curve first enters the solution region (as k increases/decreases)

IS A CORNER POINT! $x = 14$, $y = 0$

FOR US, ALL BOUNDARIES ARE STRAIGHT LINES,
AND ALL OBJECTIVE FUNCTIONS ARE LINEAR.

THEOREM 1 Fundamental Theorem of Linear Programming

If the optimal value of the objective function in a linear programming problem exists, then that value must occur at one or more of the corner points of the feasible region.

WHERE TO LOOK FOR SOL'N.

Theorem 1 provides a simple procedure for solving a linear programming problem, *provided that the problem has an optimal solution—not all do*. In order to use Theorem 1, we must know that the problem under consideration has an optimal solution. Theorem 2 provides some conditions that will ensure that a linear programming problem has an optimal solution.

THEOREM 2 Existence of Optimal Solutions

(A) If the feasible region for a linear programming problem is bounded, then both the maximum value and the minimum value of the objective function always exist.
(B) If the feasible region is unbounded and the coefficients of the objective function are positive, then the minimum value of the objective function exists but the maximum value does not.
(C) If the feasible region is empty (that is, there are no points that satisfy all the constraints), then both the maximum value and the minimum value of the objective function do not exist.

KNOW WHETHER SOL'NS EXIST.

WHAT IF THE OBJECTIVE FUNCTION IS NON-LINEAR?

ex. MAXIMIZE $z = \underbrace{3xy^2 - x^3 - y^3 - x}_{\text{EXPONENTS } \neq 1 \Rightarrow \text{ NON-LINEAR}}$

SUBJECT TO CONSTRAINTS:

$0 \leq x \leq 6$, $0 \leq y \leq 4$.

THEN MAX/MIN'S DO NOT HAVE TO BE LOCATED

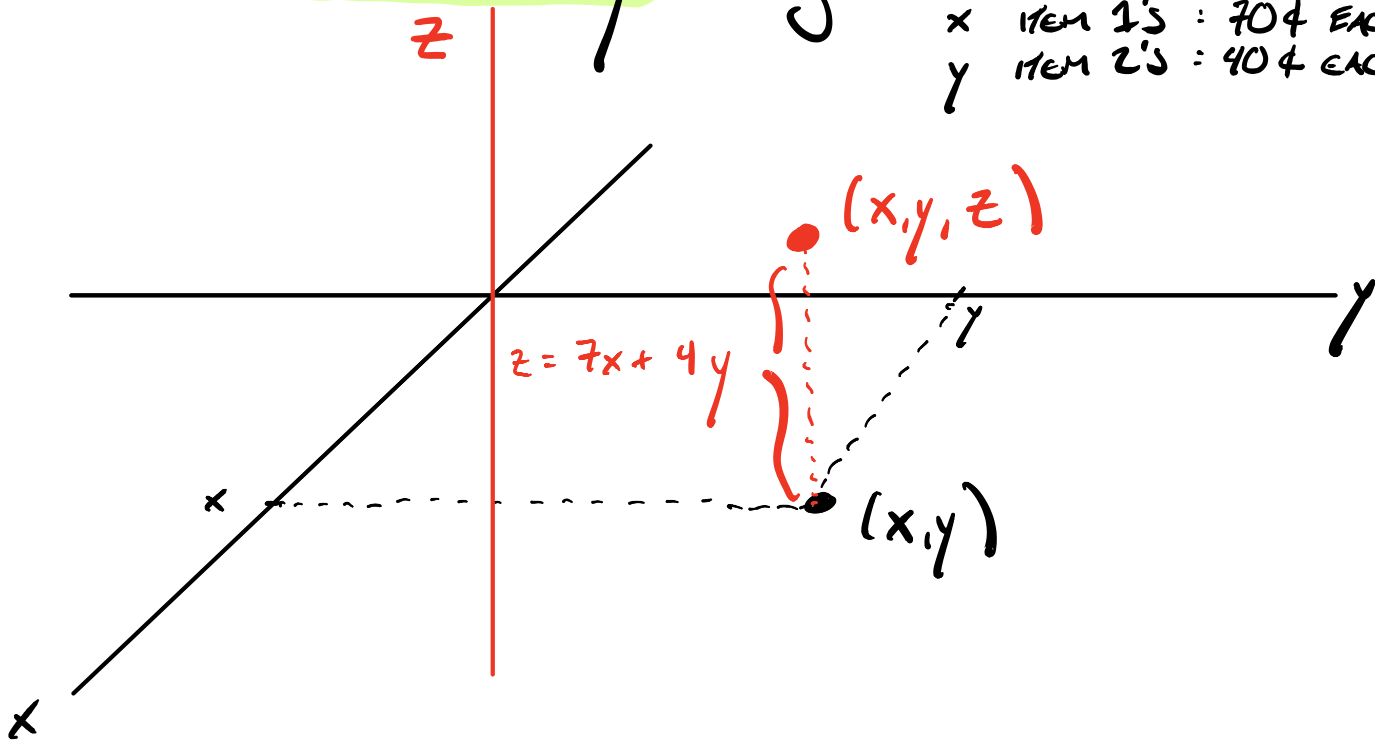
A1 Corner Points.

- They can happen at other points on the boundary of the solution region.

Graphs of Functions of 2 Variables (Objective Function)

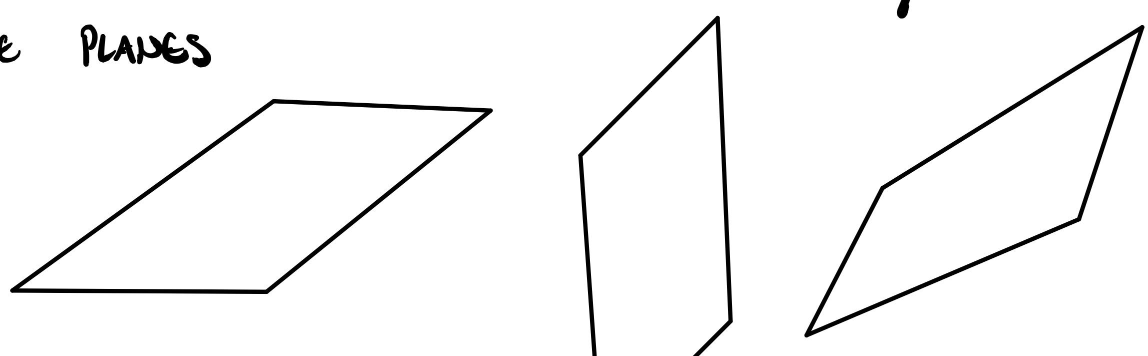
Suppose $z = 7x + 4y$

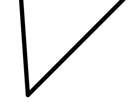
e.g. Cost of producing
x item 1's : 70¢ each
y item 2's : 40¢ each



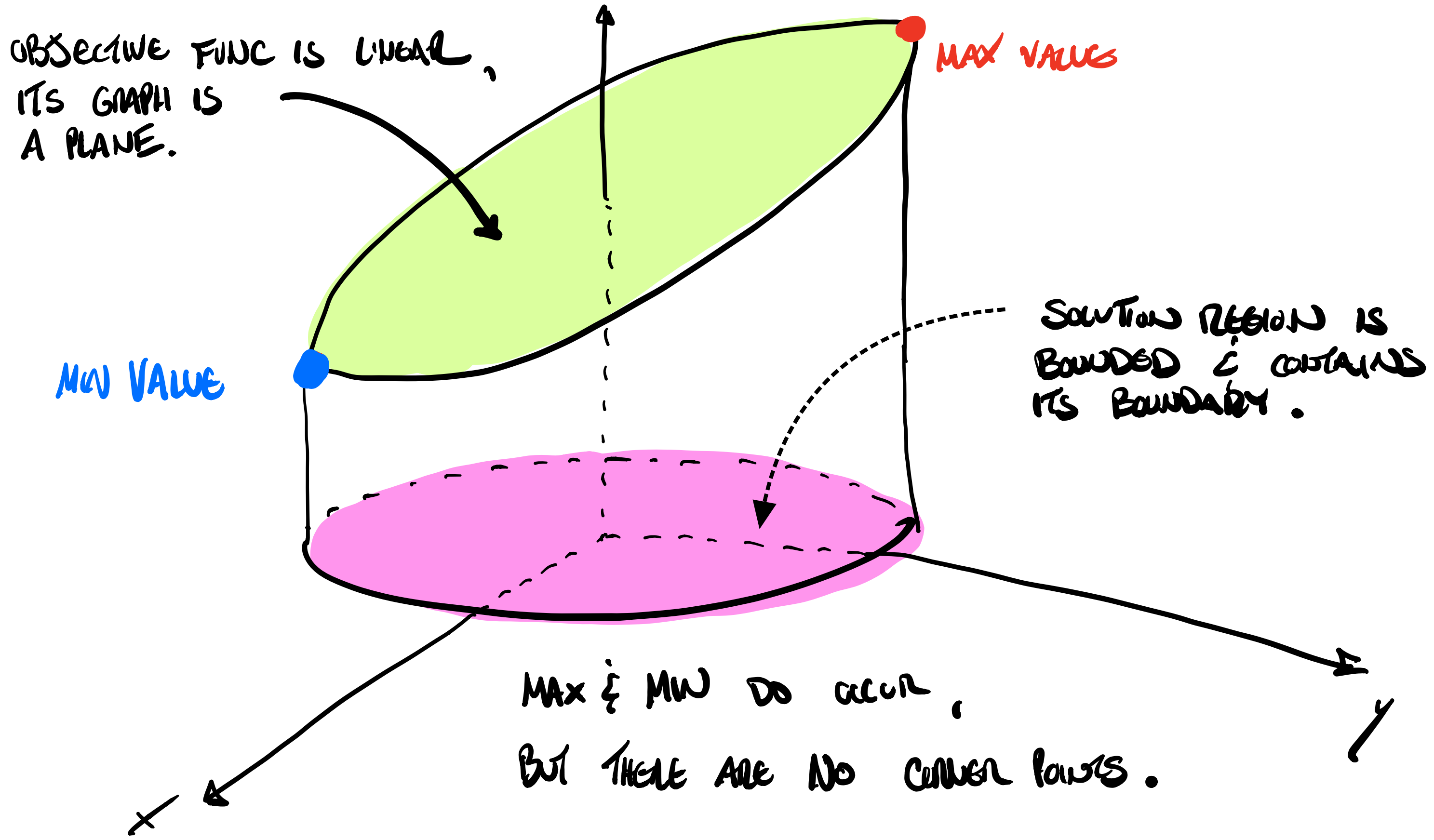
The height of the graph above (or below) the point (x, y) is the value of the function for those values of x & y.

The graphs of linear functions $z = Ax + By$ are planes





WHAT IF THE BOUNDARY IS NOT COMPOSED OF STRAIGHT LINES?



CHALLENGE: A FARMER HAS 100 ft OF FENCING & WANTS TO BUILD A RECTANGULAR FENCE THAT ENCLOSES THE LARGEST AREA POSSIBLE.
WHAT ARE THE DIMENSIONS?

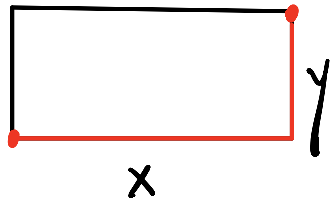
OBJECTIVE FUNCTION: MAXIMIZE AREA

$$A = xy$$

x = LENGTH
y = WIDTH

CONSTRAINTS:

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 50$$



$2x + 2y \le 100$

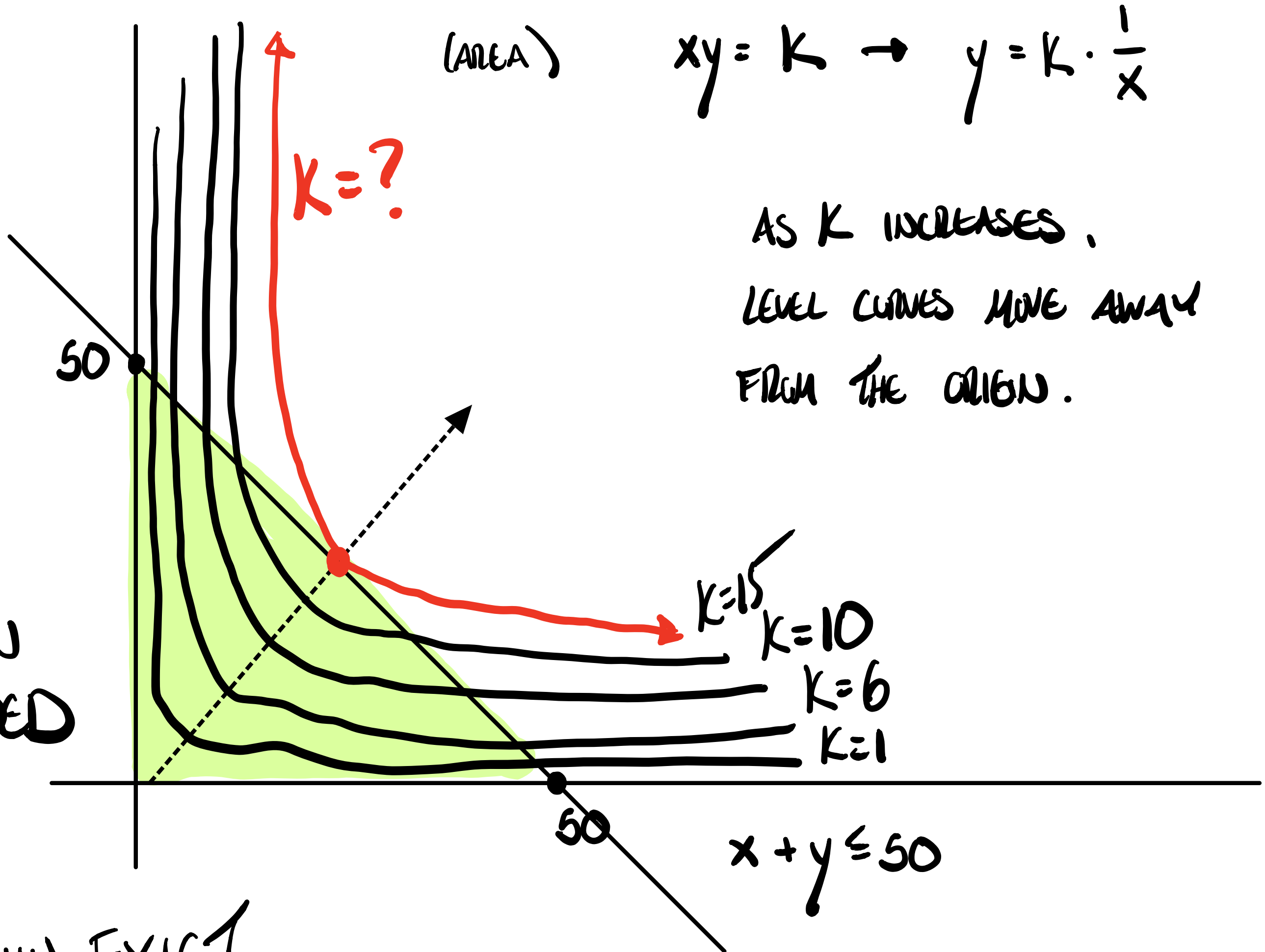
$x + y \le 50$

(1) GRAPH SOLUTION REGION.

(2) CONSIDER LEVEL CURVES :

(AREA) $xy = K \rightarrow y = K \cdot \frac{1}{x}$

AS K INCREASES, LEVEL CURVES MOVE AWAY FROM THE ORIGIN.



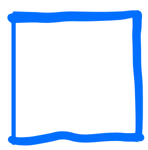
SOL'N REGION BOUNDED

$\Downarrow$

MAX/MIN EXIST.

APPROXIMATELY MAX VALUE $\approx$ 620-630

SOL'N : $x = y = 25$



$A = (25)(25) = \underline{625}$