Module 3: Simple Interest & Compound Interest

Sequence (list of numbers)

Absolute +/−

$P_0 + K = P_1$

$(K = P_1 - P_0)$

Relative ×/÷

$P_0 \times r = P_1$

$\left(r = \frac{P_1}{P_0}\right)$

Linear Growth

$$P_0 \xrightarrow{+K} P_1 \xrightarrow{+K} P_0 + 2K \xrightarrow{+K} P_0 + 3K \xrightarrow{+K} P_0 + 4K \to$$

Exponential Growth

$$P_0 \xrightarrow{\times r} P_1 \xrightarrow{\times r} P_0 r^2 \xrightarrow{\times r} P_0 r^3 \to P_0 r^4 \to$$

Accounts earning simple interest grow linearly.

Accounts earning compound interest grow exponentially.

1. Simple interest

Used by default for short-term loans/investments.

- I = interest
- P = principal
- r = annual interest rate (decimal)
- t = time (years)
- A = account balance/future value

$$A = P + Prt = P(1+rt)$$

$$\boxed{I = Prt} \qquad \boxed{A = P + I = \cancel{P(1+rt)}}$$

51. How much interest will you have to pay for a 60-day loan of \$500, if a 36% annual rate is charged?

$$I = Prt \longrightarrow I = \underbrace{(500)(.36)}_{\text{interest in 1 yr}} \frac{60}{360} = 30$$

56. A check for \$3,097.50 was used to retire a 5-month \$3,000 loan. What annual rate of interest was charged?

$$A = P(1+rt)$$

$$3097.50 = 3000\left(1 + r \cdot \frac{5}{12}\right)$$

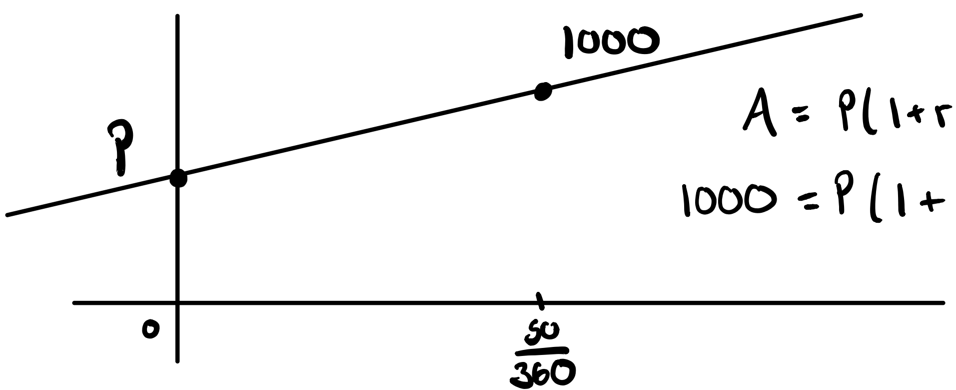
$$\frac{3097.50}{3000} = 1 + r \cdot \frac{5}{12}$$

$$\frac{3097.50}{3000} - 1 = r \cdot \frac{5}{12}$$

$$\frac{12}{5}\left(\frac{3097.50}{3000} - 1\right) = r$$

$$r = .078 \rightarrow 7.8\%$$

63. What is the purchase price of a 50-day T-bill with a maturity value of \$1,000 that earns an annual interest rate of 5.53%?

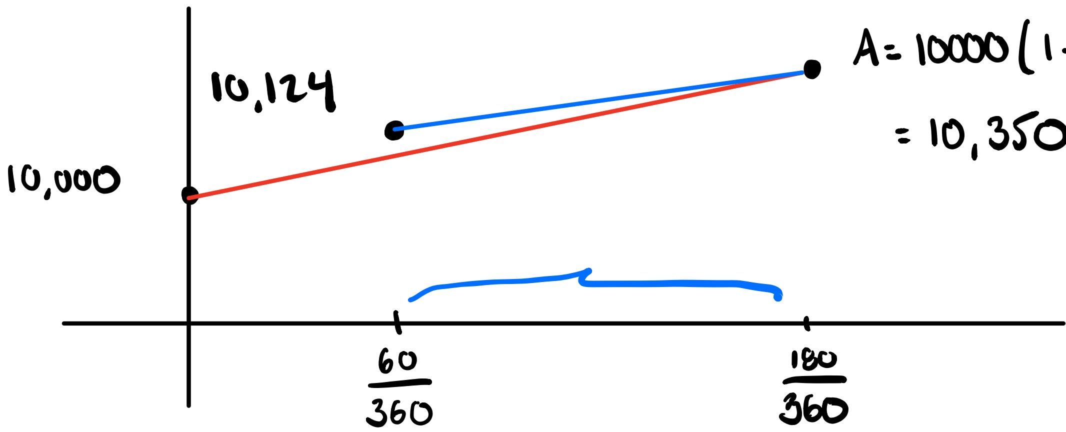


$$A = P(1+rt)$$

$$1000 = P\left(1 + .0553 \cdot \frac{50}{360}\right)$$

$$P = \frac{1000}{1 + .0553\left(\frac{50}{360}\right)} = \$992.38$$

70. To complete the sale of a house, the seller accepts a 180-day note for \$10,000 at 7% simple interest. (Both interest and principal are repaid at the end of 180 days.) Wishing to use the money sooner for the purchase of another house, the seller sells the note to a third party for \$10,124 after 60 days. What annual interest rate will the third party receive for the investment?



$$A = 10000\left(1 + .07 \times \frac{180}{360}\right)$$
$$= 10,350$$

$A = P(1+rt)$ 3rd Party:

$$10350 = 10,124\left(1 + r \times \frac{120}{360}\right)$$

$$\left(\frac{10,350}{10,124} - 1\right)\frac{360}{120} = r \approx .067$$

6.7%

- P = principal
- r = annual interest rate (decimal)
- n = number of compound periods per year
- t = time (years)
- A = account balance/compound amount
- r_E = effective rate/annual percentage yield (APY)

$$\boxed{A = P\left(1+\frac{r}{n}\right)^{nt} = P\,(1+r_E)^t}$$

$$\boxed{r_E = \left(1+\frac{r}{n}\right)^{n} - 1}$$

66. A person with \$14,000 is trying to decide whether to purchase a car now, or to invest the money at 6.5% compounded semiannually and then buy a more expensive car. How much will be available for the purchase of a car at the end of 3 years?

$$A = P\left(1+\frac{r}{n}\right)^{nt} \quad = P\left(1+\frac{r}{m}\right)^{n}$$

$$A = 14000\left(1+\frac{.065}{2}\right)^{(2)(3)}$$

$$= 16,961.66$$

70. In a suburb, housing costs have been increasing at 5.2% per year compounded annually for the past 8 years. A house worth \$260,000 now would have had what value 8 years ago?

$$A = P\left(1+\frac{r}{n}\right)^{nt}, \quad n=1$$

$$260,000 = P(1+.052)^{8}$$

$$P = \frac{260,000}{(1+.052)^{8}} = 173,319.50$$

Logarithms

IF YOU UNDERSTAND EXPONENTS
THEN YOU UNDERSTAND LOGARITHMS

$3^2 = 9$

BASE 3

$\boxed{\log_3} 9 = 2$

"Log BASE 3"

THE POWER x SUCH THAT
$3^x = 9$

$4^{1/2} = 2$

$\log_4 2 = \frac{1}{2}$

CHANGE OF BASE FORMULA

$$\log_b a = \frac{\log_c a}{\log_c b}$$

FOR ANY
$c > 0$
$c \neq 1$

e.g. $\log_{1.7} 3.8 = \frac{\log 3.8}{\log 1.7} = \frac{\ln 3.8}{\ln 1.7}$

$\log \equiv \log_{10}$

$\ln = \log_e$

$e \approx 2.718281828\ldots$

75. You have saved \$7,000 toward the purchase of a car costing \$9,000. How long will the \$7,000 have to be invested at 9% compounded monthly to grow to \$9,000? (Round up to the next-higher month if not exact.)

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$n = 12$
$nt = 12t$ = # MONTHS.

$$9000 = 7000\left(1+\frac{.09}{12}\right)^{nt}$$

$$\frac{9000}{7000} = 1.0075^{nt}$$

$b^y = x$
$\log_b x = y$

EQUIVALENT

$$1.0075^{nt} = \frac{9}{7}$$

$$\log_{1.0075} \frac{9}{7} = nt$$

$$nt = \log_{1.0075} \frac{9}{7} = \frac{\ln(9/7)}{\ln(1.0075)}$$

$$= 33.63$$

34 MONTHS

88. What is the annual nominal rate compounded monthly for a bond that has an annual percentage yield of 2.95%?

PRINCIPAL EARNING INTEREST RATE r COMPOUNDED n TIMES PER YEAR.

$P = 100$

$r = 10\%$

$n = 12$

AFTER 1 YEAR: $A = 100\left(1+\frac{.10}{12}\right)^{12}$

$= 110.47$

EFFECTIVE (GROWTH) RATE / ANNUAL PERCENTAGE YIELD (APY)

10.47%

NOMINAL RATE

$$A = P(1+r_E)^t \qquad A = P\left(1+\frac{r}{n}\right)^{nt}$$

SET EQUAL $\cancel{P}(1+r_E)^{\cancel{t}} = \cancel{P}\left(1+\frac{r}{n}\right)^{n\cancel{t}}$

$$1+r_E = \left(1+\frac{r}{n}\right)^n$$

$$(1+r_E)^{1/n} = 1+\frac{r}{n}$$

$$(1+r_E)^{1/n} - 1 = \frac{r}{n}$$

$$n\left[(1+r_E)^{1/n} - 1\right] = r$$

$$12\left[(1+.0295)^{1/12} - 1\right] = r = .0291$$

2.91%

The buying and selling commission schedule shown in the table is from an online discount brokerage firm. Taking into consideration the buying and selling commissions in this schedule, find the annual compound rate of interest earned by each investment in Problems 95–98.

Transaction Size	Commission Rate
\$0–\$1,500	\$29 + 2.5% of principal
\$1,501–\$6,000	\$57 + 0.6% of principal
\$6,001–\$22,000	\$75 + 0.30% of principal
\$22,001–\$50,000	\$97 + 0.20% of principal
\$50,001–\$500,000	\$147 + 0.10% of principal
\$500,001+	\$247 + 0.08% of principal

97. An investor purchases 200 shares of stock at \$28 per share, holds the stock for 4 years, and then sells the stock for \$55 a share.

98. An investor purchases 400 shares of stock at \$48 per share, holds the stock for 6 years, and then sells the stock for \$147 a share.

97.

INVEST: $\overbrace{200 \times 28}^{5,600}$

$+ 57 + .006(5600)$

$= 5690.60$

VALUE AFTER 4 YEARS:

200×55

$11,000 - 75 - .003(11,000)$

$= 10,892$

$$A = P(1+r)^t$$

$$10,892 = 5690.6(1+r)^4$$

$$\left(\frac{10,892}{5690.6}\right)^{1/4} - 1 = r = .1762$$

17.62%