Module 3: Simple Interest & Compound Interest

T-Bill maturity value \$2032 (50 day)
Annual interest rate 3.886% (assume 360 day year).

Under 1 year: default simple interest

$I = Prt$, $A = P + I$

$A = P + Prt$

$A = P(1 + rt)$

$$\frac{A}{1+rt} = P = \frac{2032}{1 + .03886\left(\frac{50}{360}\right)}$$

$= 2021.0917\ldots$

$\rightarrow$ \$2021.09

Tip: Do not round until you arrive at the final answer!

Linear vs. Exponential Growth:

Sequence

Absolute +/-

$P_1 = P_0 + K$

$(K = P_1 - P_0)$

P_0

+K $\downarrow$ $\quad$ $\downarrow$ $\times r$

P_1

+K $\downarrow$ $\quad$ $\downarrow$ $\times r$

$P_2 = P_0 + 2K$ $\quad$ $P_2 = P_0 r^2$

+K $\quad$ $\times r$

Relative ×/÷

$P_1 = P_0 \cdot r$

$\left(r = \frac{P_1}{P_0}\right)$

$P_3 = P_0 + 3k$

$+k$

$$\boxed{P_n = P_0 + nk}$$

LINEAR GROWTH

ACCOUNTS EARNING SIMPLE INTEREST GROW LINEARLY

APPLY TO FINANCE

$P_3 = P_0 r^3$

$\times r$

$$\boxed{P_n = P_0 r^n}$$

EXPONENTIAL GROWTH

ACCOUNTS EARNING COMPOUND INTEREST GROW EXPONENTIALLY

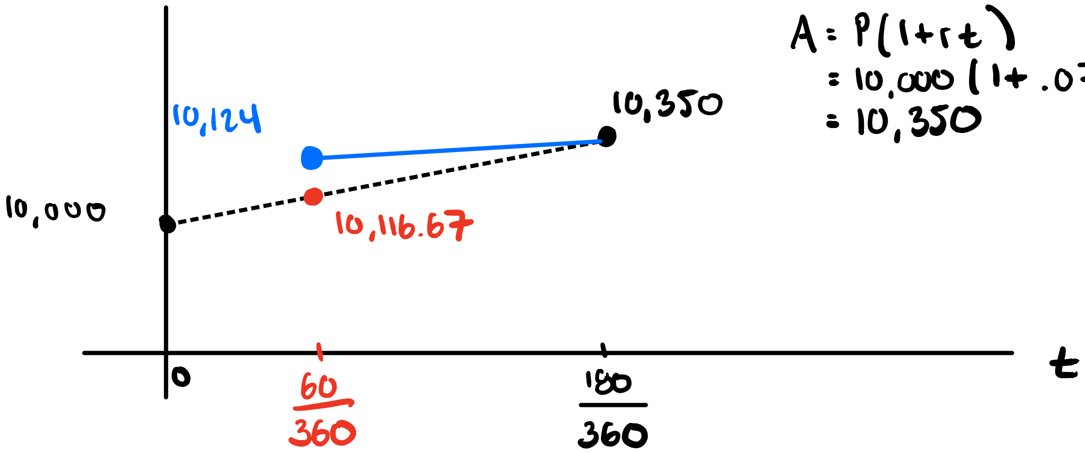
1. Simple interest

Used by default for short-term loans/investments.

- I = interest
- P = principal
- r = annual interest rate (decimal)
- t = time (years)
- A = account balance/future value

$$\boxed{I = Prt} \qquad \boxed{A = P + I = P(1 + r)t}$$

70. To complete the sale of a house, the seller accepts a 180-day note for \$10,000 at 7% simple interest. (Both interest and principal are repaid at the end of 180 days.) Wishing to use the money sooner for the purchase of another house, the seller sells the note to a third party for \$10,124 after 60 days. What annual interest rate will the third party receive for the investment?



$$A = P(1+rt)$$
$$= 10{,}000\left(1+.07\left(\tfrac{1}{2}\right)\right)$$
$$= 10{,}350$$

$$A = 10{,}000\left(1+.07\left(\tfrac{1}{6}\right)\right) = 10{,}116.67$$

3rd PARTY: PRINCIPAL 10,124 FUTURE VALUE: 10,350

TIME : $\frac{120}{360}$.

$$A = P(1+rt)$$

$$10{,}350 = 10{,}124\left(1+r\left(\tfrac{120}{360}\right)\right)$$

$$\frac{10350}{10{,}124} = 1 + \frac{r}{3}$$

$$\frac{10350}{10{,}124} - 1 = \frac{r}{3}$$

$$3\left(\frac{10350}{10{,}124} - 1\right) = r = .066969$$

$$\rightarrow .067 \quad \boxed{6.7\%}$$

2. Compound interest

- P = principal
- r = annual interest rate (decimal)
- n = number of compound periods per year
- t = time (years)
- A = account balance/compound amount
- r_E = effective rate/annual percentage yield (APY)

$$\boxed{A = P\left(1+\frac{r}{n}\right)^{nt} = P\left(1+r_E\right)^t} \qquad \boxed{r_E = \left(1+\frac{r}{n}\right)^n - 1}$$

$$A = P\underbrace{\left[\left(1+\frac{r}{n}\right)^n\right]}_{\text{BASE}}^t$$

← EXPONENT IN YEARS

IF $t = 1$:

$$\left(1+\frac{r}{n}\right)^n$$

EVERY YEAR, MONEY IS MULTIPLIED BY THIS NUMBER

$$r_E = \left(1+\frac{r}{n}\right)^n - 1$$

$$A = P\underbrace{\left[1+\frac{r}{n}\right]}_{\text{BASE}}^{nt}$$

← # COMPOUND PERIODS

$$2 \xrightarrow{\times 1.5} 3$$

INCREASES BY 13% ⟷ MULTIPLIED BY 1.13

$$3 = 2 + \underline{.5}(2) = \underbrace{2(1+\underline{.5})}_{\text{50\% GROWTH}}$$

$$2 \xrightarrow{\times 2} 4$$

$$4 = 2 + \underline{100\%}\,2$$

100% GROWTH.

66. A person with $14,000 is trying to decide whether to purchase a car now, or to invest the money at 6.5% compounded semiannually and then buy a more expensive car. How much will be available for the purchase of a car at the end of 3 years?

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 14000\left(1+\frac{.065}{2}\right)^{(2)(3)}$$

$$= \$16961.66$$

70. In a suburb, housing costs have been increasing at 5.2% per year compounded annually for the past 8 years. A house worth $260,000 now would have had what value 8 years ago?

$$A = P\left(1+\frac{r}{n}\right)^{nt} \rightarrow P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$$

$$P = \frac{260{,}000}{\left(1+\frac{.052}{1}\right)^{(1)(8)}} = \boxed{\$173{,}319.50}$$

LOGARITHMS

ANOTHER WAY TO EXPRESS EXPONENTIAL RELATIONS BETWEEN NUMBERS

$$3^2 = 9 \qquad \log_3 9 = 2$$

"HE IS MY FATHER" "I AM HIS SON"

$$\boxed{\begin{array}{c} b^x = a \\ \Updownarrow \\ \log_b a = x \end{array}}$$

"LOG-BASE-3 OF 9 IS 2"

(EXPONENT)

$\log_b a$ IS THE # YOU WOULD RAISE b TO IN ORDER TO EQUAL a.

ex. $\log_2 8 = 3 \iff 2^3 = 8$

$\log_{25} 5 = x \iff 25^x = 5$

$x = \frac{1}{2}$ $\qquad 25^{1/2} = \sqrt{25} = 5$

$$\boxed{\begin{array}{c} b^x = a \\ \Updownarrow \\ \log_b a = x \end{array}}$$

ex. $1.06^x = 1.74 \iff \log_{1.06} 1.74 = x$

$$\boxed{\begin{array}{c} b^x = a \\ \Updownarrow \\ \log_b a = x \end{array}}$$

CHANGE OF BASE FORMULA:

$$\log_A B = \frac{\log_c B}{\log_c A}$$

FOR ANY $c > 0$, $c \neq 1$.

e.g. $\log_A B = \dfrac{\log_{10} B}{\log_{10} A}$

ON MOST CALC $\log = \log_{10}$

$$\log_A B = \frac{\log_e B}{\log_e A} \qquad \log_e = \ln$$

$e = 2.718281828$

75. You have saved \$7,000 toward the purchase of a car costing \$9,000. How long will the \$7,000 have to be invested at 9% compounded monthly to grow to \$9,000? (Round up to the next-higher month if not exact.)

(SOLVE FOR TIME : EXPONENT)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$9000 = 7000\left(1 + \frac{.09}{12}\right)^{12t}$$

$$\boxed{1.0075^{12t} = \frac{9}{7}}$$

$$\boxed{b^x = a \iff \log_b a = x}$$

$$\log_{1.0075} \frac{9}{7} = 12t$$

\# MONTHS

$$\frac{\ln \frac{9}{7}}{\ln 1.0075} = 12t = 33.63\ldots$$

$\rightarrow$ $\boxed{34 \text{ MONTHS}}$

OR $\log_{10}$ OR ANY LOGARITHM AT ALL

The buying and selling commission schedule shown in the table is from an online discount brokerage firm. Taking into consideration the buying and selling commissions in this schedule, find the annual compound rate of interest earned by each investment in Problems 95–98.

Transaction Size	Commission Rate
\$0–\$1,500	\$29 + 2.5% of principal
\$1,501–\$6,000	\$57 + 0.6% of principal
\$6,001–\$22,000	\$75 + 0.30% of principal
\$22,001–\$50,000	\$97 + 0.20% of principal
\$50,001–\$500,000	\$147 + 0.10% of principal
\$500,001+	\$247 + 0.08% of principal

97. An investor purchases 200 shares of stock at \$28 per share, holds the stock for 4 years, and then sells the stock for \$55 a share.

98. An investor purchases 400 shares of stock at \$48 per share, holds the stock for 6 years, and then sells the stock for \$147 a share.

PRINCIPAL: $P = \underbrace{200 \times 28}_{5600} + 57 + .006(5600)$

$P = \$5690.60$

ACCOUNT BALANCE: $A = \underbrace{200 \times 55}_{11,000} - 75 - .003(11,000)$

$= \$10,892$

(n=1 COMPOUND/YEAR): $A = P(1+r)^t$

$10,892 = 5690.60(1+r)^4$

$\frac{10892}{5690.6} = (1+r)^4$

$$\left[\frac{10892}{5690.6}\right]^{1/4} = \left[(1+r)^{\cancel{4}}\right]^{1/\cancel{4}}$$

$$\left[\frac{10892}{5690.6}\right]^{1/4} = 1+r$$

$$r = \left[\frac{10892}{5690.6}\right]^{1/4} - 1 = .1762168\ldots$$

$$= 17.62\%$$ ANNUAL INTEREST.