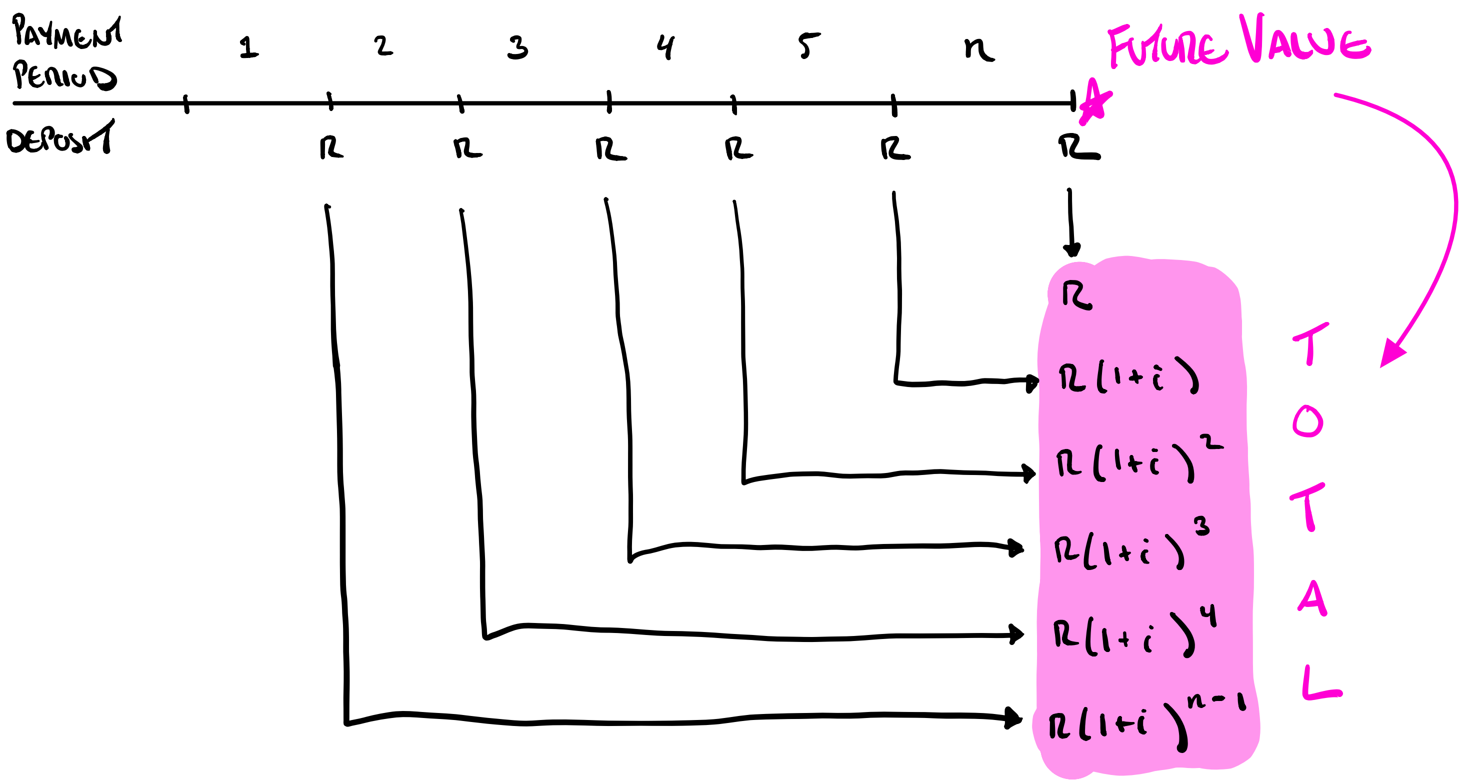
§3.3 Future Value of an Annuity

An ordinary annuity is a sequence of equal size payments R deposited at the end of each payment period into an account earning an interest rate i per payment period, compounded at the end of each payment period.

Suppose you make n payments.



Let S_n be the future value.

$$S_n = R + R(1+i) + R(1+i)^2 + \dots + R(1+i)^{n-1}$$ (Geometric Series)

$$(1+i)S_n = R(1+i) + R(1+i)^2 + \dots + R(1+i)^{n-1} + R(1+i)^n$$

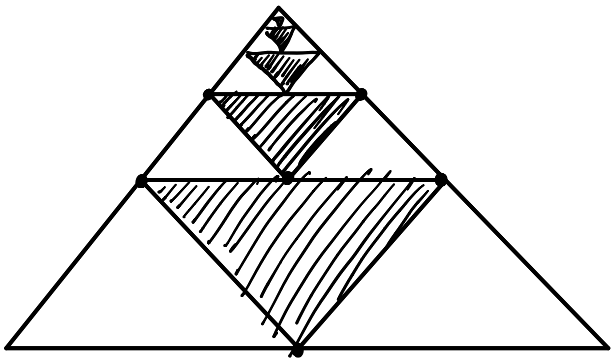
$$(1+i)S_n - S_n = R(1+i)^n - R$$

$$iS_n = R\left[(1+i)^n - 1\right]$$

$$\boxed{S_n = \frac{R\left[(1+i)^n - 1\right]}{i}}$$

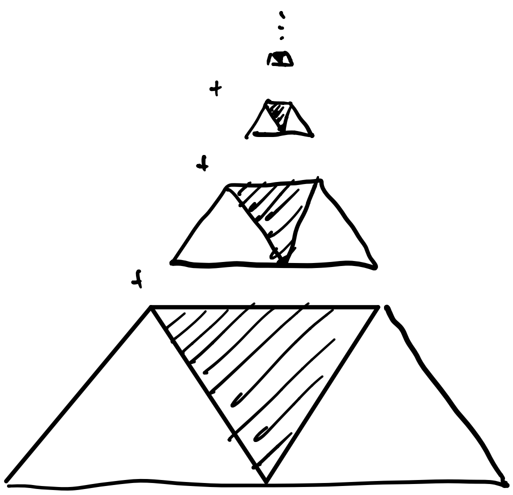
ANOTHER GEOMETRIC SERIES:

EQUILATERAL TRIANGLE WITH AREA 1. AREA OF SHADED REGION

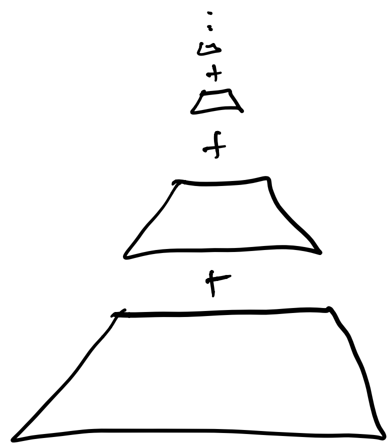


$$\frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^4 + \left(\frac{1}{4}\right)^5 + \ldots =$$

GEOMETRIC SERIES



$= \frac{1}{3}$ [



]

$$= \frac{1}{3}(1) = \frac{1}{3}$$

28. USG Annuity and Life offered an annuity that pays 7.25% compounded monthly. If $1,000 is deposited into this annuity every month, how much is in the account after 15 years? How much of this is interest?

$$S_n = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$S_n = \frac{1000\left[\left(1+\frac{.0725}{12}\right)^{180} - 1\right]}{\frac{.0725}{12}}$$

$$= \$323,943.07$$

180 DEPOSITS OF $1,000

$180,000

$$\text{INTEREST} = S_n - nR = \$143,943.07$$

43. Compubank, an online banking service, offered a money market account with an APY of 1.551%.

(A) If interest is compounded monthly, what is the equivalent annual nominal rate?

(B) If you wish to have $10,000 in this account after 4 years, what equal deposit should you make each month?

$$r_E = \left(1+\frac{i}{n}\right)^n - 1$$

i = NOMINAL RATE

n = # COMPOUNDS PER YEAR

$$.01551 = \left(1+\frac{i}{12}\right)^{12} - 1$$

$$1.01551 = \left(1+\frac{i}{12}\right)^{12}$$

$$(1.01551)^{1/12} = 1 + \frac{i}{12}$$

$$12\left(1.01551^{1/12} - 1\right) = i = .01540$$

NOMINAL RATE

ANNUAL INTEREST RATE

INTEREST RATE PER PAYMENT PERIOD.

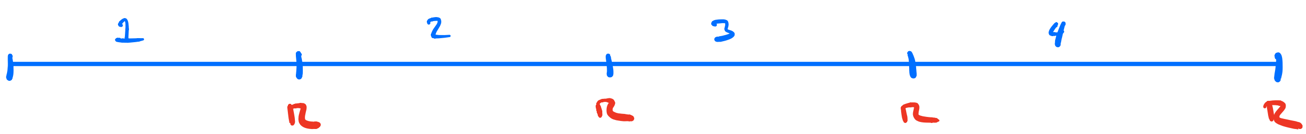
(b) $$S_n = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$10000 = \frac{R\left[\left(1+\frac{.01540}{12}\right)^{48} - 1\right]}{\frac{.01540}{12}}$$

SOLVE FOR R.

ex. IF \$2000 IS DEPOSITED EVERY 6 MONTHS INTO AN ACCOUNT EARNING 6% INTEREST COMPOUNDED SEMIANNUALLY FOR 2 YEARS, CONSTRUCT A BALANCE SHEET FOR THE INTEREST EARNED AND THE ACCOUNT BALANCE AT THE END OF EACH 6 MONTH PERIOD.

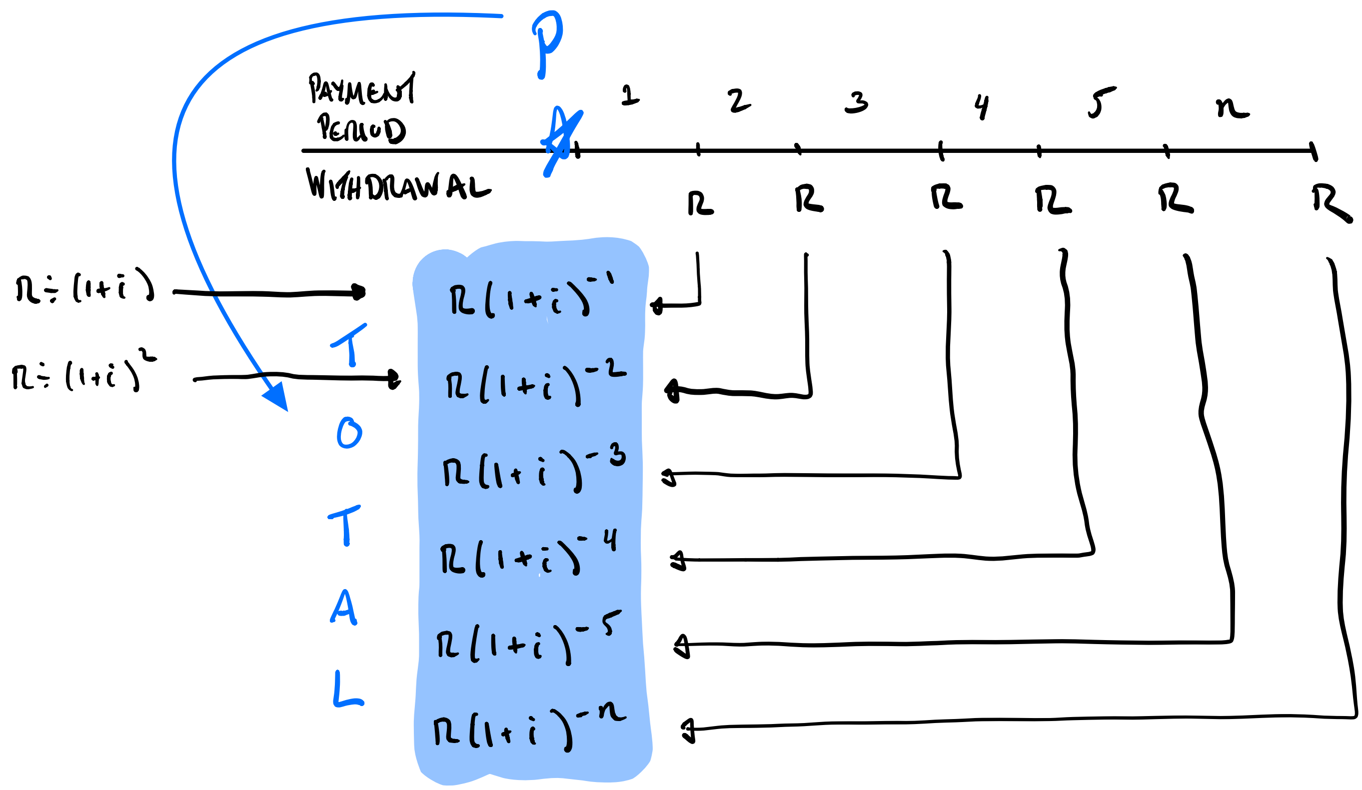
PERIOD	DEPOSIT	INTEREST	BALANCE
1	2000	0 3%	2000
2	2000	60 3%	4060
3	2000	121.80 3%	6181.80
4	2000	185.45	8367.25



§3.4 Present Value of an Annuity

Now suppose you want to make n withdrawals of equal size R at the end of each payment period from an account earning interest rate i per payment period, compounded at the end of each payment period.

Let P be the amount of money you would need to deposit today so that after making these withdrawals your account balance is \$0.



$$P = R(1+i)^{-1} + R(1+i)^{-2} + \dots + R(1+i)^{-(n-1)} + R(1+i)^{-n}$$

$$(1+i)P = R + R(1+i)^{-1} + \dots + R(1+i)^{-(n-2)} + R(1+i)^{-(n-1)}$$

$$(1+i)P - P = R - R(1+i)^{-n}$$

$$iP = R\left(1 - (1+i)^{-n}\right)$$

$$\boxed{P = \frac{R\left[1-(1+i)^{-n}\right]}{i}}$$

$$S_n = \frac{R\left[(1+i)^n - 1\right]}{i}$$

42. A recreational vehicle costs $80,000. You pay 10% down and amortize the rest with equal monthly payments over a 7-year period. If you pay 9.25% compounded monthly, what is your monthly payment? How much interest will you pay?

10% OF 80,000 = 8,000

$72,000 LOAN AT $\frac{.0925}{12}$ INTEREST PER MONTH.

$$P = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$72{,}000 = \frac{R\left[1-\left(1+\frac{.0925}{12}\right)^{-84}\right]}{\frac{.0925}{12}}$$

$$R = \frac{72000\left(\frac{.0925}{12}\right)}{\left[1-\left(1+\frac{.0925}{12}\right)^{-84}\right]} = \$1{,}167.57$$

TOTAL PAYMENTS: $nR = 84(1{,}167.57)$

$= \$98{,}075.88$

44. Construct the amortization schedule for a $10,000 debt that is to be amortized in six equal quarterly payments at 2.6% interest per quarter on the unpaid balance.

Period	Payment	Interest	Reduction	Balance
0	-	-	-	10,000
1	1821.58	260	1561.58	8,438.42
2	1821.58	219.40	1602.18	6836.24
3	1821.58	177.74	1643.84	5192.40
4	1821.58	135.00	1686.58	3505.82
5	1821.58	91.15	1730.43	1775.39
6	~~1821.58~~ 1821.55	46.16	1775.42	~~.03~~ 0

×.026

2.6%

$$P = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$R = \frac{Pi}{\left[1-(1+i)^{-n}\right]} = \frac{10000(.026)}{\left[1-(1.026)^{-6}\right]}$$

$$= \$1,821.58$$