$f(x) = x + \sqrt{8-x}$

$g(u) = u + \sqrt{8-u}$

IS $f = g$ YES ✓

$(\quad) \longmapsto (\quad) + \sqrt{8-(\quad)}$

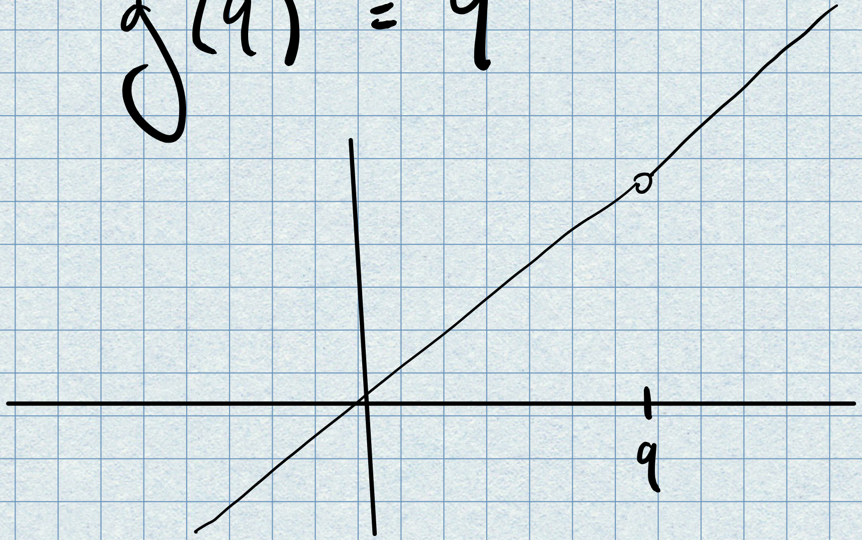
$f(x) = \dfrac{x^2 - 9x}{x-9}$ $\qquad g(x) = x$

IS $f = g$? $\quad \hookrightarrow \dfrac{x\cancel{(x-9)}}{\cancel{(x-9)}} = x$

$f(x) = g(x)$ AGREE ON THEIR DOMAINS

HOWEVER $f(9)$ IS UNDEFINED

$g(9) = 9$



FINISH §1.1, SKIP 1.2, BRIEFLY 1.3

START 1.4 (CALCULUS BEGINS) & HW#2

---

§1.1 CONTINUED.

PIECEWISE DEFINED FUNCTIONS

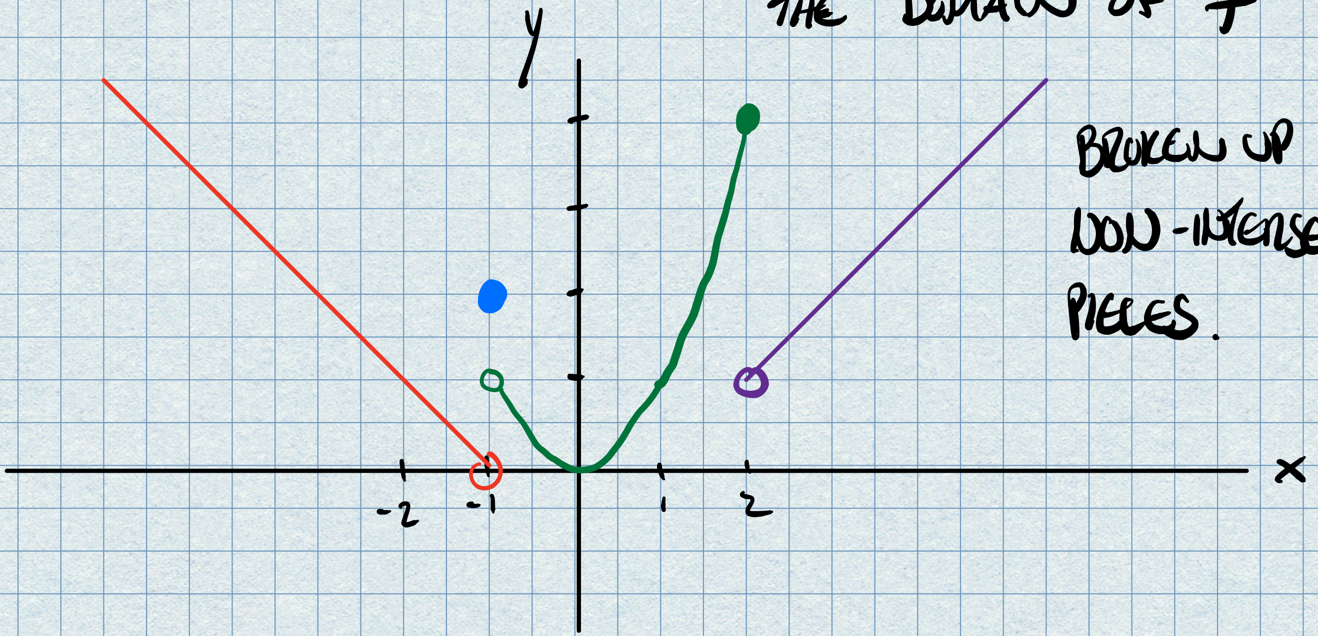
WE WILL FOLLOW DIFFERENT RULES FOR DIFFERENT INPUT

ex.

$$f(x) = \begin{cases} -x-1 & \text{IF } x < -1 & (1) \\ 2 & \text{IF } x = -1 & (2) \\ x^2 & \text{IF } -1 < x \le 2 & (3) \\ x-1 & \text{IF } x > 2 & (4) \end{cases}$$

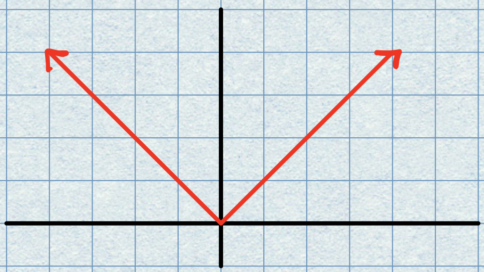
CONDITIONS PARTITION THE DOMAIN OF f

BROKEN UP INTO NON-INTERSECTING PIECES.



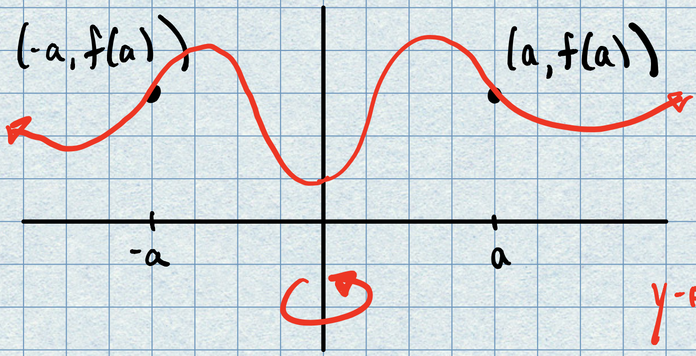
Def: THE ABSOLUTE VALUE FUNCTION

$$f(x) = |x| = \begin{cases} x & \text{IF } x \geq 0 \\ -x & \text{IF } x < 0 \end{cases}$$



SYMMETRY:

Def: FUNC. f IS EVEN IF $f(-x) = f(x)$ FOR ALL x IN $\mathrm{Dom}(f)$



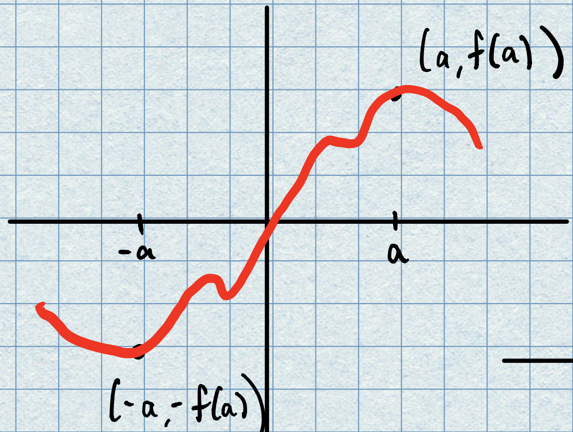
← OUTPUTS EQUAL

(EVEN)

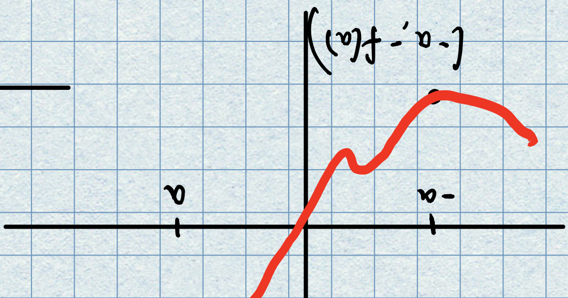
y-AXIS IS LINE OF SYMMETRY

OPPOSITE INPUTS

Def: FUNC f IS ODD IF $f(-x) = -f(x)$ FOR ALL x IN $\mathrm{Dom}(f)$

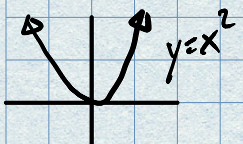


GRAPH IS UNCHANGED WHEN ROTATED 180°

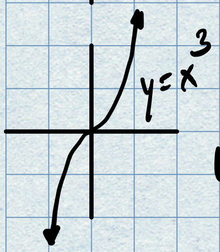


((0)f'(0))

ex. $f(x) = x^2$ IS EVEN : $f(-x) = (-x)^2 = (-1)^2 x^2 = x^2 = f(x)$ ✓



$g(x) = x^3$ IS ODD : $f(-x) = (-x)^3 = (-1)^3 x^3 = -x^3 = -f(x)$ ✓



IN GENERAL $P(x) = x^n$ POWER FUNCTION
IS EVEN IF n IS EVEN &
ODD IF n IS ODD.

PROPERTIES: FOR EXAMPLE IF f, g EVEN FUNC.
r, s ODD FUNC.

$r(x)s(x)$ EVEN OR ODD? ex: $x^3 x^5 = x^8$ EVEN

$$r(-x)s(-x) = [-r(x)][-s(x)] = (-1)^2 r(x)s(x) = r(x)s(x)$$

EVEN!

ie (ODD)(ODD) = EVEN

OTHER PROP: (EVEN)(ODD) = ODD
(EVEN)(EVEN) = EVEN

$\dfrac{\text{ODD}}{\text{ODD}}$ = EVEN

ETC...

ex. ex: $f(x) = \dfrac{x^3 + x^5}{x^2 + 4} = \dfrac{\text{ODD} + \text{ODD}}{\text{EVEN} + \text{EVEN}}$

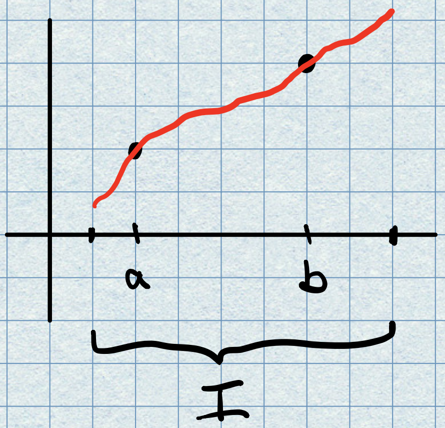
$= \frac{\text{ODD}}{\text{EVEN}} = \text{ODD}$

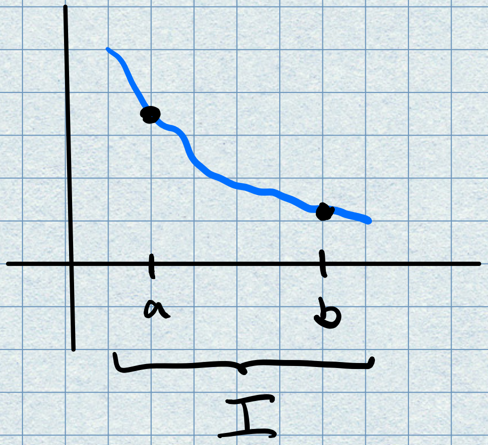
INCREASING / DECREASING

Def: A FUNCTION f IS INCREASING (DECREASING) ON AN INTERVAL I

IF FOR ANY $a < b$ IN I WE HAVE

$f(a) < f(b)$ $\qquad$ $f(a) > f(b)$

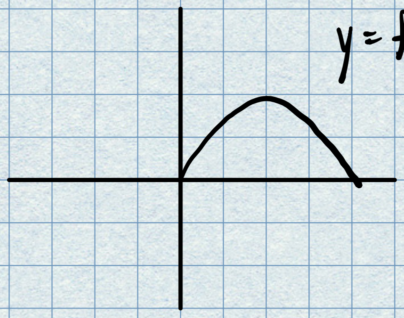




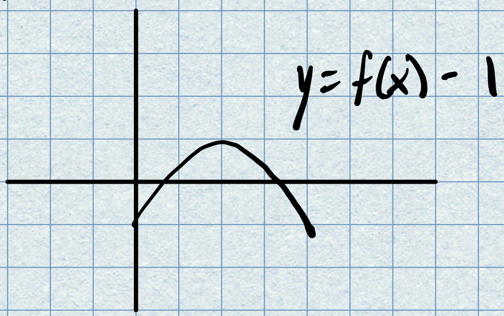
§1.3 NEW FUNCTIONS FROM OLD FUNCTIONS

TRANSFORMATIONS:

$y = f(x)$



$y = f(x) + c$ $\quad$ VERTICAL SHIFT BY c

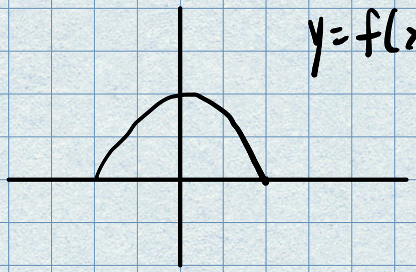


$y = f(x + c)$ $\quad$ HORIZONTAL SHIFT BY c

LEFT IF $c > 0$

RIGHT IF $c < 0$

$y = f(x+2)$

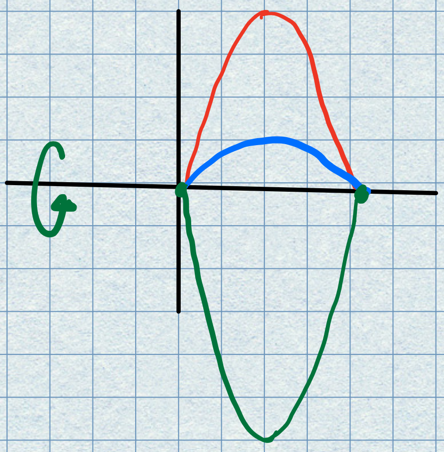


$y = a\,f(x)$ VERTICAL SCALING BY FACTOR OF a

STRETCH FROM X-AXIS IF $|a| > 1$

SHRINK TOWARD X-AXIS IF $|a| < 1$

REFLECT ACROSS X-AXIS IF $a < 0$



$y = 2f(x)$

$y = \frac{1}{2} f(x)$

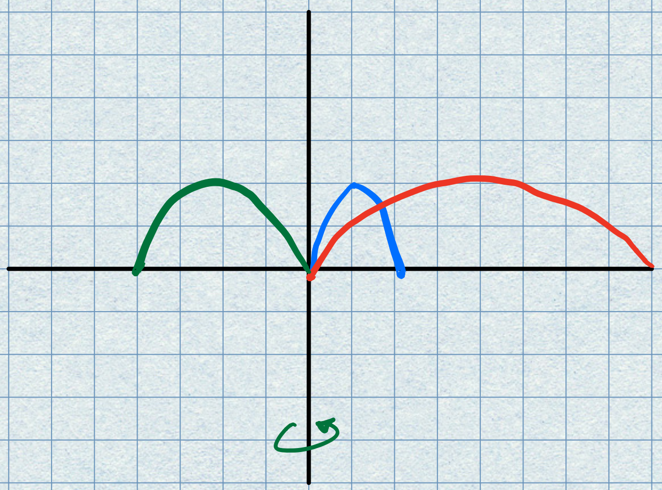
$y = -3 f(x)$

$y = f(ax)$ HORIZONTAL SCALING BY FACTOR OF a

STRETCH ABOUT Y-AXIS IF $|a| < 1$

SHRINK TOWARD Y-AXIS IF $|a| > 1$

REFLECT ACROSS Y-AXIS IF $a < 0$



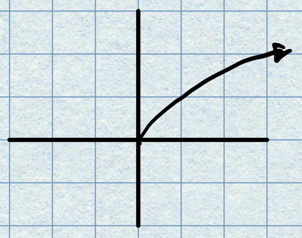
$y = f(2x)$

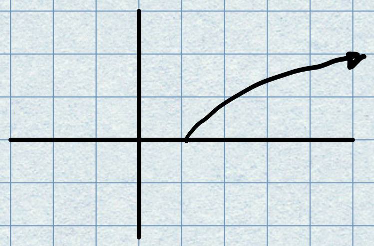
$y = f(\frac{1}{2}x)$

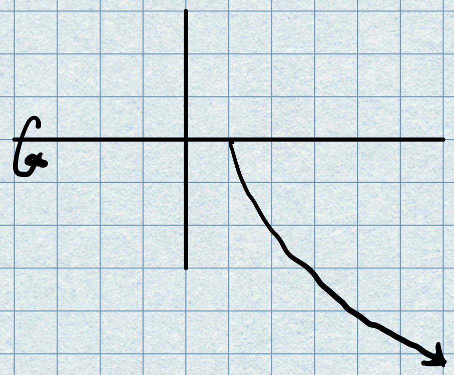
$y = f(-x)$

ex. GRAPH $f(x) = -2\sqrt{x-1}$

$$y=\sqrt{x} \longrightarrow y=\sqrt{x-1} \longrightarrow y=-2\sqrt{x-1}$$







COMPOSITION OF FUNCTIONS

NOTATION: $f \circ g(x) = f(g(x))$

$$x \longmapsto \boxed{g} \longrightarrow g(x) \longrightarrow \boxed{f} \longrightarrow f(g(x))$$

ex. $f(x) = 3x - 2$

$g(x) = x^2 - 1$

i) $f(g(2)) = \begin{cases} f(3) \\ 3(g(2)) - 2 \end{cases} =$

$$= \begin{cases} 3(3) - 2 \\ 3(3) - 2 \end{cases} = \boxed{7}$$

ex. $f(x) = \sqrt{1+x}$

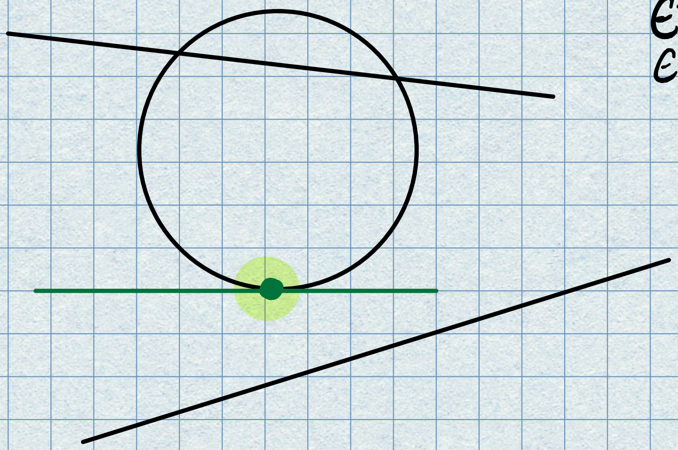
FIND $f \circ f(x) = f(f(x))$

$$= \sqrt{1 + f(x)}$$

$$= \sqrt{1 + \sqrt{1+x}}$$

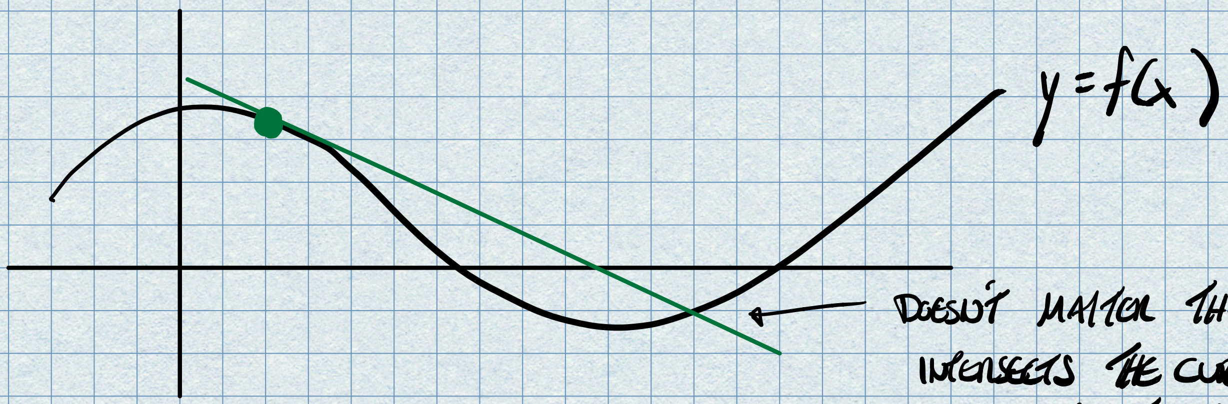
§1.4 THE TANGENT & VELOCITY PROBLEMS

GEOMETRIC DEF OF TANGENT LINE:



EVERY LINE INTERSECTS A CIRCLE
EITHER 0 TIMES
1 TIMES ← TANGENT LINE TO A CIRCLE
2 TIMES

NOW WE EXTEND / GENERALIZE TO DEFINE TANGENT LINES TO OTHER CURVES.



DOESN'T MATTER THAT IT INTERSECTS THE CURVE MORE THAN ONCE.

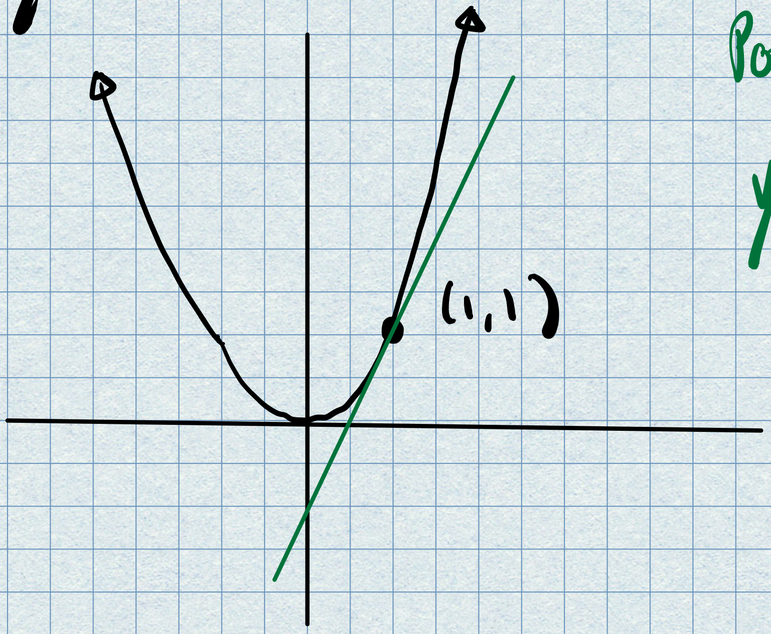
IF WE ZOOM IN ON THIS CURVE AT THIS POINT, THE CURVE BEGINS TO APPEAR TO FLATTEN.

IT LOOKS LIKE A STRAIGHT LINE

TANGENT LINE

FINDING TANGENT LINES:

FIND EQ OF THE LINE TANGENT TO $y = x^2$ AT $(1,1)$.



POINT-SLOPE FORMULA

$$y - 1 = m(x - 1)$$

FIND SLOPE m