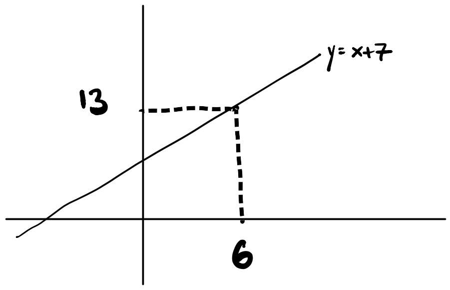
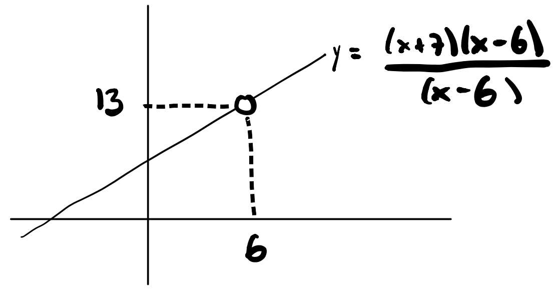




$$\lim_{x\to 6} x+7 = 13$$

$$\lim_{x\to 6} \frac{(x+7)(x-6)}{(x-6)} = 13$$

THM. IF $f(x) = g(x)$ FOR x NEAR a, EXCEPT POSSIBLY AT a, THEN $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$.

$$\lim_{x\to 49} \frac{7-\sqrt{x}}{49x - x^2}$$

PLUG IN: $\frac{0}{0}$

NEEDS WORK

$$\lim_{x\to 49} \frac{7-\sqrt{x}}{49x-x^2}\left(\frac{7+\sqrt{x}}{7+\sqrt{x}}\right)$$

$$(a-b)(a+b) = a^2 - b^2$$

$$= \lim_{x\to 49} \frac{(7)^2 - (\sqrt{x})^2}{x(49-x)(7+\sqrt{x})}$$

$$= \lim_{x\to 49} \frac{\cancel{49-x}}{x\cancel{(49-x)}(7+\sqrt{x})}$$

ASSUMING $x \neq 49$

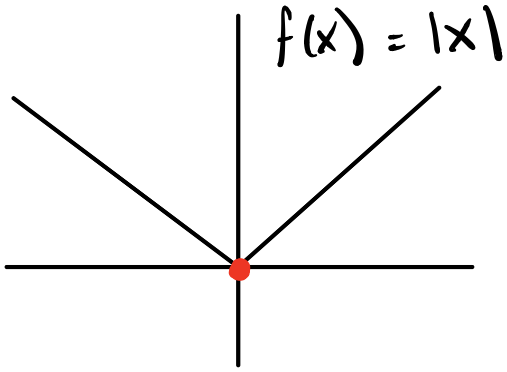
$$= \lim_{x\to 49} \frac{1}{x(7+\sqrt{x})} = \frac{1}{49(7+\sqrt{49})} = \frac{1}{49 \times 14}$$

§2.2 THE DERIVATIVE AS A FUNCTION

$f'(x)$ GIVES THE DERIVATIVE OF f AT THE POINT x,

FOR ALL x SUCH THAT $\underbrace{\text{THE DERIVATIVE OF } f}_{\text{A LIMIT}}$ EXISTS.

$\mathrm{Dom}(f') \subseteq \mathrm{Dom}(f)$



CORNER

$f'(0)$ DNE

$$f'(0) = \lim_{h \to 0} \frac{|0+h| - |0|}{h}$$

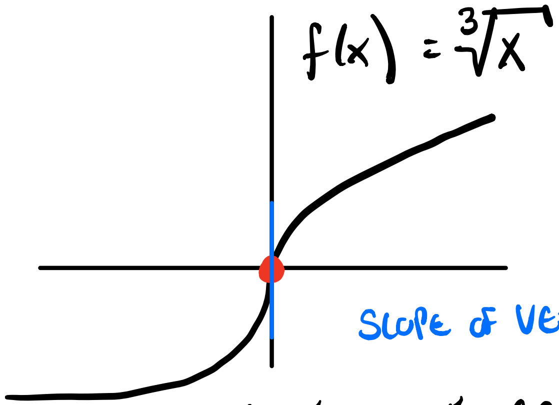
RECALL $|x| = \begin{cases} x & \text{IF } x \geq 0 \\ -x & \text{IF } x < 0 \end{cases}$

IF $h < 0$ $\quad \lim_{h \to 0^-} \frac{|h|}{h} = \lim_{h \to 0^-} \frac{-h}{h} = -1$

IF $h > 0$ $\quad \lim_{h \to 0^+} \frac{|h|}{h} = \lim_{h \to 0^+} \frac{h}{h} = 1$

SINCE $\lim_{h \to 0^-}$ & $\lim_{h \to 0^+}$ ARE DIFFERENT,

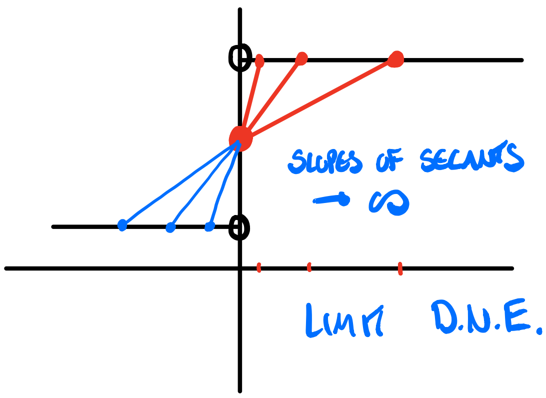
THE LIMIT $\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$ DNE.



SLOPE OF VERTICAL LINES IS NOT DEFINED

VERTICAL TANGENT

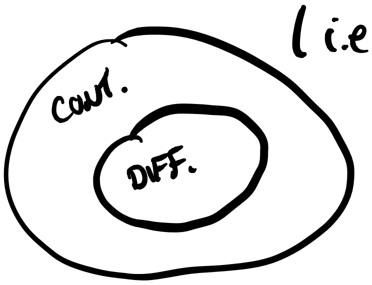
$f'(0)$ D.N.E. (SLOPE OF TANGENT LINE AT $x=0$)



$$f(x) = \begin{cases} 3 & \text{IF } x > 0 \\ 2 & \text{IF } x = 0 \\ 1 & \text{IF } x < 0 \end{cases}$$

$f'(0)$ D.N.E. $\qquad f$ IS NOT CONTINUOUS AT $x=0$

IN FACT: IF f IS NOT CONTINUOUS AT a
THEN f IS NOT DIFFERENTIABLE AT a
(i.e. $f'(a)$ D.N.E.)



EQUIVALENT

THEOREM: IF f IS DIFFERENTIABLE AT a
THE f IS CONTINUOUS AT a.

" DIFFERENTIABILITY IS MORE THAN

PROOF: WE MUST SHOW $\lim_{x \to a} f(x) = f(a)$

i.e. $\lim_{x \to a} [f(x) - f(a)] = 0$. CONTINUITY. "

WE HAVE

$$\lim_{x \to a} [f(x) - f(a)] = \lim_{x \to a} \left[\frac{f(x) - f(a)}{x - a} (x - a) \right]$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot \lim_{x \to a} (x - a)$$

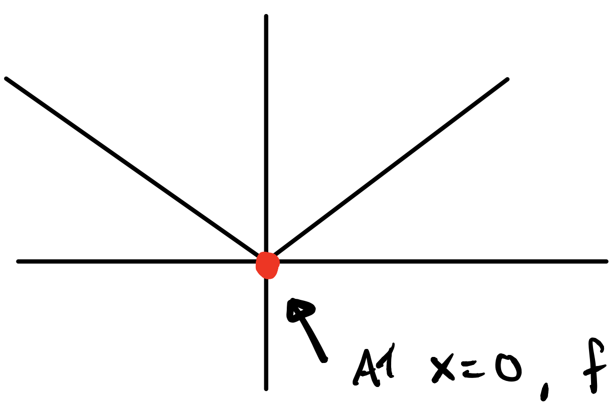
$$= f'(a) \cdot 0 = 0. \quad \square$$

f IS CONTINUOUS AT a : GRAPH OF f HAS NO HOLES, NO JUMPS, NO ASYMPTOTES

"CONNECTED"

f IS DIFFERENTIABLE AT a : AS WE ZOOM IN, THE GRAPH APPEARS TO BE MORE & MORE LIKE A STRAIGHT LINE WITH A DEFINED SLOPE.

"SMOOTH"



$f(x) = |x|$ IS CONTINUOUS ON $(-\infty, \infty)$ & DIFFERENTIABLE ON $(-\infty, 0) \cup (0, \infty)$.

AT $x = 0$, $f(x) = |x|$ IS CONTINUOUS BUT NOT DIFFERENTIABLE

OTHER NOTATIONS

LET f BE A DIFFERENTIABLE FUNCTION OF x.

LET $y = f(x)$. THE FOLLOWING ARE EQUIVALENT   DEE DEE EX

$$f'(x) = y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x)$$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \begin{matrix} \leftarrow \Delta y \\ \leftarrow \Delta x \end{matrix}$$

"THE DERIVATIVE OF"

(AN OPERATOR ON THE FUNCTION f)

$$= D_x f(x) = D f(x)$$

AND

$$f'(a) = \frac{dy}{dx}\bigg|_{x=a} = \left.\frac{dy}{dx}\right]_{x=a}$$

VERTICAL BAR = "EVALUATE AT"

HIGHER ORDER DERIVATIVES

FUNCTION $f(x)$. LET $y = f(x)$.

(FIRST) DERIVATIVE $f'(x) = y' = \frac{d}{dx} f = \frac{dy}{dx}$

SECOND DERIVATIVE = DERIVATIVE OF DERIVATIVE

$$f''(x) = (f'(x))' = y'' = \frac{d}{dx}\left(\frac{d}{dx} f\right)$$

$$= \frac{d^2}{dx^2} f = \frac{d^2 y}{dx^2}$$

dx^2 $\qquad$ dx^2

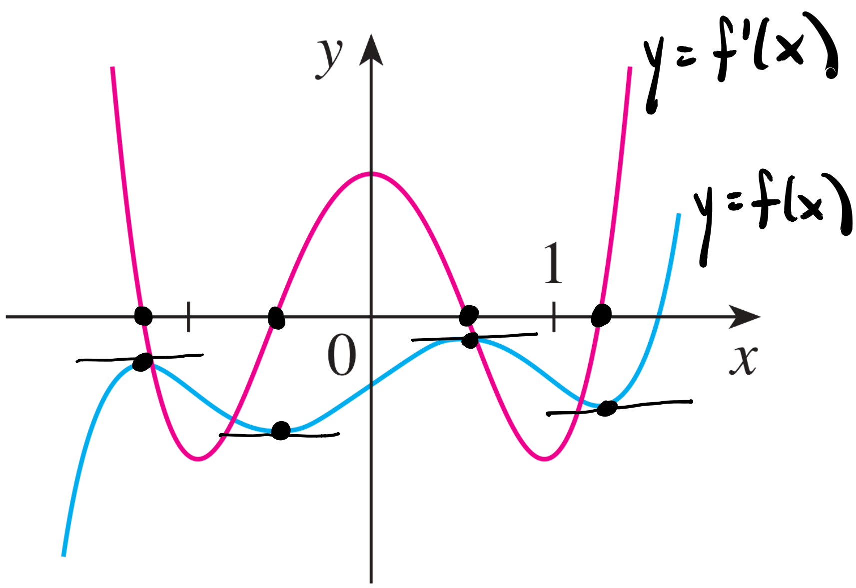
THIRD DERIVATIVE = DERIVATIVE OF 2nd DERIVATIVE.

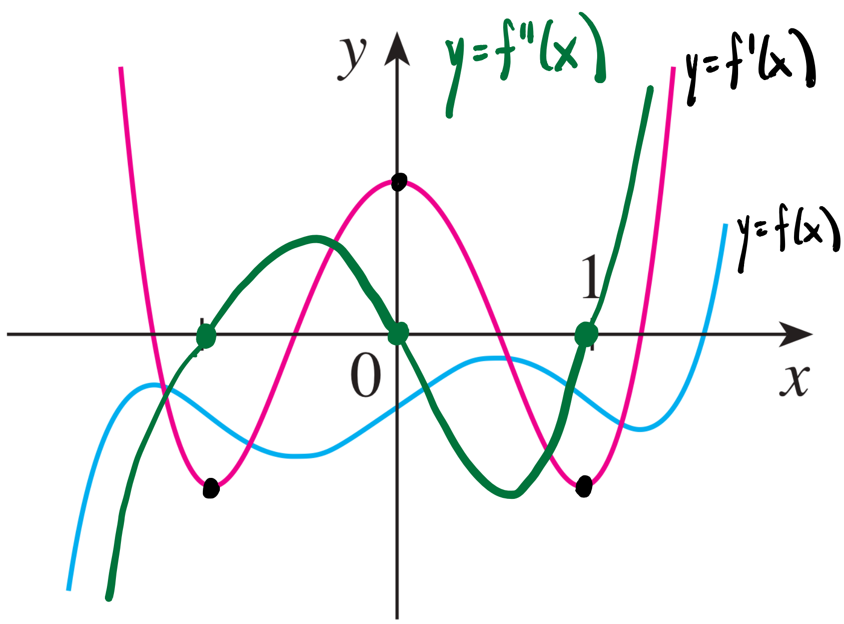
$$f'''(x) = y''' = \frac{d^3}{dx^3} f = \frac{d^3 y}{dx^3}$$

n^{TH} DERIVATIVE : $f^{(n)}(x)$, $n \geq 4$

$$f''''''(x) = f^{(6)}(x)$$

x





LET $s(t)$ GIVES THE POSITION OF AN OBJECT ALONG A STRAIGHT PATH

$s'(t) = v(t)$ VELOCITY OF OBJECT

$s''(t) = v'(t) = a(t)$ ACCELERATION OF OBJECT

$s'''(t) = v''(t) = a'(t) = j(t)$ "JERK"

$j'(t) =$ "SNAP"

§2.3 DIFFERENTIATION RULES

LET c BE A CONSTANT.

$$\boxed{\frac{d}{dx} c = 0}$$

PROOF: $\underset{\substack{\uparrow \\ f(x)=c}}{\frac{d}{dx} c} = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} 0 = 0.$

POWER RULE:

LET n BE A POSITIVE INTEGER $(n = 1, 2, 3, \ldots)$.

$$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$$

BINOMIAL FORMULA

PROOF:

$$\frac{d}{dx}x^n = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \dots n x h^{n-1} + h^n \cancel{- x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(n x^{n-1} + h(\dots)\right)}{\cancel{h}}$$

$$= \lim_{h \to 0} n x^{n-1} + h(\dots) = n x^{n-1} \qquad \square$$

e.g. $\frac{d}{dx} x^9 = 9x^{9-1} = 9x^8$.