$y = x^5 - 5x \xrightarrow{\text{x-INT.}} x^5 - 5x = 0$

$x(x^4 - 5) = 0$

$x(x^2 + \sqrt{5})(x^2 - \sqrt{5}) = 0$

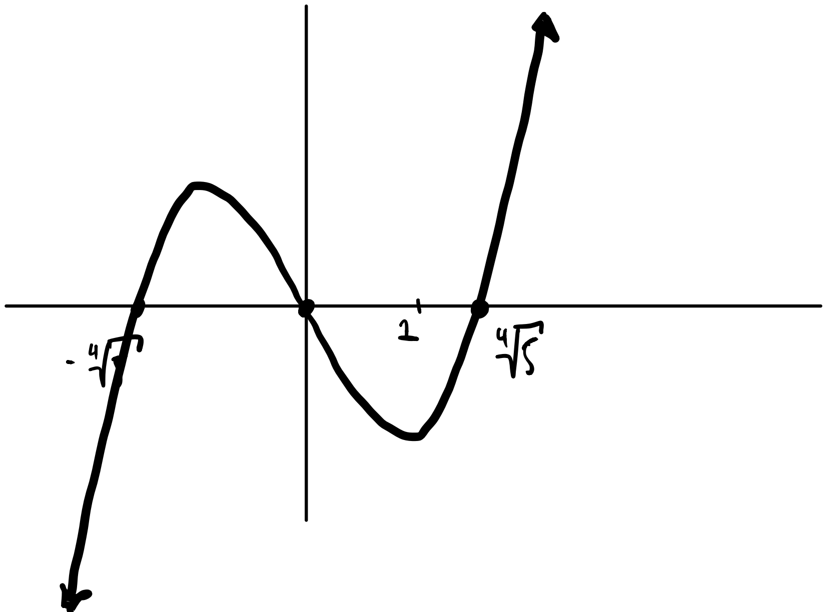
$x(x^2 + \sqrt{5})(x + \sqrt[4]{5})(x - \sqrt[4]{5}) = 0$

$x = 0$ $\quad \neq 0$ $\quad x = -\sqrt[4]{5}$ $\quad x = \sqrt[4]{5}$

DOM = $\mathbb{R}$

POLYNOMIAL WITH ONLY ODD DEGREE TERMS:

ODD SYMMETRY



$f'(x) = 5x^4 - 5 = 0$

$5(x^4 - 1) = 0$

$5(x^2+1)(x+1)(x-1) = 0$

$\neq 0$ $\quad x = -1$ $\quad x = 1$

	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
f	↗	↘	↗
f'	+	−	+
$5(x^2+1)$	+	+	+
$(x+1)$	−	+	+
$(x-1)$	−	−	+

$f(x) = x^5 - 5x$

$f(-x) = (-x)^5 - 5(-x)$

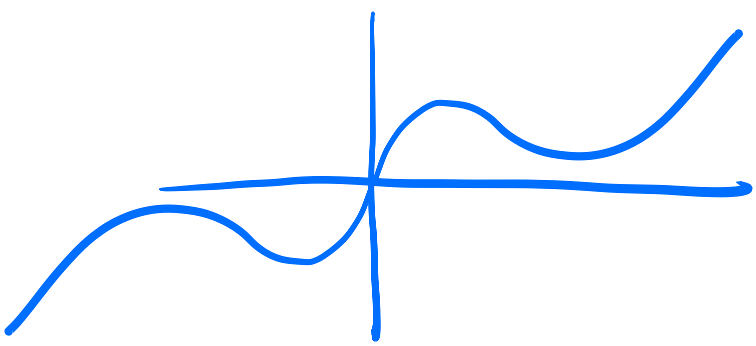
$= (-1)^5 x^5 + 5x$

$= -x^5 + 5x = -(x^5 - 5x) = -f(x)$

$f(2) = 32 - 10 = 22$

$f(-2) = -32 + 10 = -22$

$f(-x) = -f(x)$



Def: LET $F(x)$ & $f(x)$ BE TWO FUNCTIONS DEFINED ON AN INTERVAL I. THE FOLLOWING STATEMENTS ARE EQUIVALENT:

f IS THE DERIVATIVE OF F ON I.

F IS AN ANTIDERIVATIVE OF f ON I.

BOTH STATEMENTS SAY $F'(x) = f(x)$ ON I.

ex. IF $f(x) = x^2$, THEN AN ANTIDERIVATIVE OF f IS $F(x) = \frac{1}{3}x^3$.

CHECK: $\frac{d}{dx}\left[F(x)\right] = \frac{1}{3}\cdot 3x^2 = x^2$ ✓

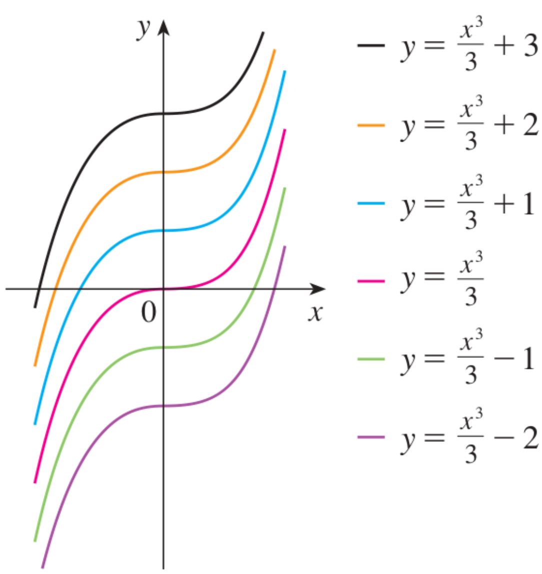
HOW ABOUT $F(x) = \frac{1}{3}x^3 + 1 \longrightarrow F'(x) = x^2$

NOTE: ANTIDERIVATIVES ARE NOT UNIQUE.

IF $\frac{d}{dx}\left[F(x)\right] = f(x)$ THEN $\frac{d}{dx}\left[F(x) + C\right] = f(x)$.

SO $\frac{1}{3}x^3$, $\frac{1}{3}x^3 + 1$, $\frac{1}{3}x^3 + \frac{13}{7}$, $\frac{1}{3}x^3 - \pi^2$, ETC.

ARE ALL ANTIDERIVATIVES OF $f(x) = x^2$.



QUESTION: ARE THERE ANY OTHERS NOT OF THE FORM $\frac{1}{3}x^3 + C$?

RECALL FROM §3.2 MEAN VALUE THM

7 Corollary If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f - g$ is constant on (a, b); that is, $f(x) = g(x) + c$ where c is a constant.

PROOF: SET $h(x) = f(x) - g(x)$

$h'(x) = f'(x) - g'(x) = 0$ BY ASSUMPTION.

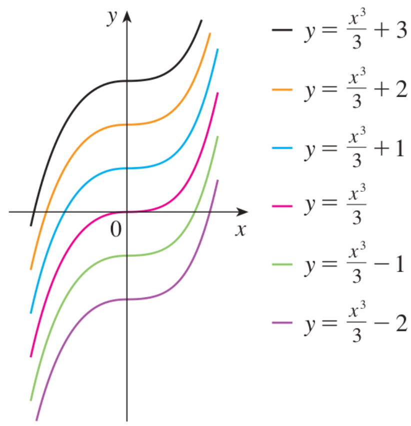
$\Rightarrow$ $h(x) = c$ CONSTANT

$\therefore$ $f(x) - g(x) = c \Rightarrow f(x) = g(x) + c$ $\square$

$\therefore$ IF F IS ONE PARTICULAR ANTIDERIVATIVE OF f THEN EVERY ANTIDERIVATIVE OF f HAS THE FORM

$F(x) + C$ FOR SOME VALUE OF C.

A "FAMILY" OF ANTIDERIVATIVES.



COMPLETE ✓

$\Rightarrow$ **1 Theorem** If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

ex. FIND THE MOST GENERAL ANTIDERIVATIVE OF

(a) $f(x) = \cos x$

$F(x) = \sin x$

CHECK: $\frac{d}{dx}[F(x)] = \frac{d}{dx}[\sin x] = \cos x = f(x)$ ✓

(b) $f(x) = -5\cos x$

$F(x) = -5\sin x$

$$\frac{d}{dx}\left[x^5\right] = 5x^4$$

$$\frac{1}{5}\frac{d}{dx}x^5 = x^4$$

$$\frac{d}{dx}\left[\frac{1}{5}x^5\right] = x^4$$

(c) $f(x) = 1 + x + x^2 + x^3 + x^4$

TERM BY TERM

$$F(x) = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5$$

(d) $f(x) = x^n \quad , \; n \geq 0$

$$F(x) = \frac{1}{n+1}x^{n+1}$$

CHECK: $\frac{d}{dx}\left[\frac{1}{n+1}x^{n+1}\right] = \frac{1}{\cancel{n+1}}\cancel{(n+1)}x^{(n+\cancel{1})-\cancel{1}}$

$= x^n$ ✓

(e) $f(x) = \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} = x^{-2} + x^{-3} + x^{-4}$

$$F(x) = -\frac{1}{x} - \frac{1}{2x^2} - \frac{1}{3x^3} + C$$

$$F(x) = -x^{-1} - \frac{1}{2}x^{-2} - \frac{1}{3}x^{-3} + C$$

CHECK: $\frac{d}{dx}\left[-x^{-1} - \frac{1}{2}x^{-2} - \frac{1}{3}x^{-3} + C\right]$

$= -(-1)x^{-2} - \frac{1}{2}(-2)x^{-3} - \frac{1}{3}(-3)x^{-4} + 0$ ✓

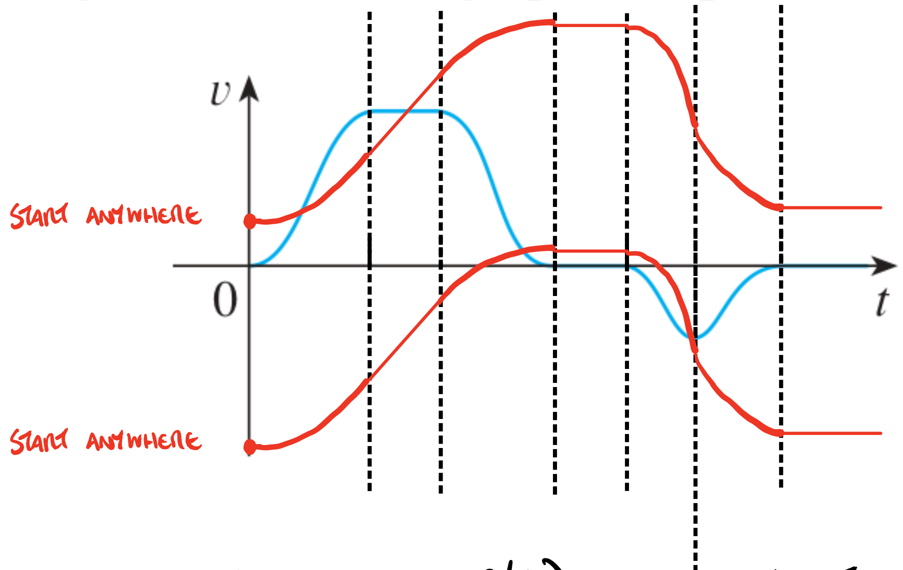
2 Table of Antidifferentiation Formulas

To obtain the most general antiderivative from the particular ones in Table 2, we have to add a constant (or constants), as in Example 1.

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\cos x$	$\sin x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sin x$	$-\cos x$
$x^n \ (n \neq -1)$ *	$\dfrac{x^{n+1}}{n+1}$	$\sec^2 x$	$\tan x$
		$\sec x \tan x$	$\sec x$

* WHAT IS AN ANTIDERIV. OF $f(x) = \frac{1}{x}$

48. The graph of the velocity function of a particle is shown in the figure. Sketch the graph of a position function.



VALUE OF $v(t)$ = SLOPE OF $s(t)$

THE DERIV. OF POSITION FUNC $s(t)$ IS VELOCITY FUNC. $v(t)$, POSITION $s(t)$ IS AN ANTIDERIV. OF VELOCITY $v(t)$.

ex. FIND $f(x)$ IF $f'(x) = \underbrace{\sqrt{x}\,(6+5x)}_{\text{TURN THIS PRODUCT INTO A SUM}}$ & $f(1) = 10$

$$f'(x) = 6\sqrt{x} + 5x\sqrt{x} = 6x^{1/2} + 5x^{3/2}$$

$$f(x) = 6\,\frac{1}{\frac{1}{2}+1}\,x^{\frac{1}{2}+1} + 5\,\frac{1}{\frac{3}{2}+1}\,x^{\frac{3}{2}+1} + C = 6\,\frac{1}{3/2}\,x^{3/2} + 5\,\frac{1}{5/2}\,x^{5/2} + C$$

$$f(x) = 6 \cdot \frac{2}{3}\,x^{3/2} + 5 \cdot \frac{2}{5}\,x^{5/2} + C$$

$$f(x) = 4x^{3/2} + 2x^{5/2} + C$$

$f(1) = 4 + 2 + C = 10 \quad \Rightarrow \quad C = 4$

$$f(x) = 4x^{3/2} + 2x^{5/2} + 4$$

ex. FIND $f(x)$ IF $f''(t) = 4 - \frac{6}{t^4}$, $f(1) = 6$, $f'(2) = 9$, $t > 0$

$f''(t) = 4 - 6t^{-4}$

$f'(t) = 4t - 6\left(\frac{1}{-4+1}\right)t^{-4+1} = \underline{4t + 2t^{-3} + C}$

$f'(2) = 4(2) + 2(2)^{-3} + C = 8 + 2 \cdot \frac{1}{2^3} + C = 9$

$8 + \frac{1}{4} + C = 9$

$C = \frac{3}{4}$

$\underline{f'(t) = 4t + 2t^{-3} + \frac{3}{4}}\left(t^0\right)$

$f(t) = 4\,\frac{1}{1+1}t^{1+1} + 2\,\frac{1}{-3+1}t^{-3+1} + \frac{3}{4}t + C$

$f(t) = 2t^2 - t^{-2} + \frac{3}{4}t + C$

$f(1) = 2(1)^2 - (1)^{-2} + \frac{3}{4}(1) + C = 6$

$2 - 1 + \frac{3}{4} + C = 6 \quad \Rightarrow \quad C = 4\frac{1}{4} = \frac{17}{4}$

$$\boxed{f(t) = 2t^2 - t^{-2} + \frac{3}{4}t + \frac{17}{4}}$$

60. Show that for motion in a straight line with constant acceleration a, initial velocity v_0, and initial displacement s_0, the displacement after time t is

$$s = \frac{1}{2}at^2 + v_0 t + s_0$$

$a(t) = a$. $v'(t) = a(t)$ => $v(t)$ IS ANTIDERIV OF $a(t)$

$v(t) = at + C$

INITIAL VELOCITY IS V_0 => $v(0) = a(0) + C = V_0$

$C = V_0$

$v(t) = at + V_0$

$s'(t) = v(t)$ => $s(t)$ IS AN ANTIDERIV. OF $v(t)$

$s(t) = a\frac{1}{2}t^2 + V_0 t + C$

$s(0) = a\frac{1}{2}(0)^2 + V_0(0) + C = S_0$

$C = S_0$

$s(t) = \frac{a}{2}t^2 + V_0 t + S_0$ ✓

$V_0 = 0$

63. A stone was dropped off a cliff and hit the ground with a speed of 120 ft/s. What is the height of the cliff?

ACCELERATION DUE TO GRAVITY $-9.8\ m/s^2$ OR $-32\ ft/s^2$

$a(t) = -32$

$v(t) = -32t + V_0$, $v(0) = 0$ => $v(t) = -32t$

$s(t) = -32 \cdot \frac{1}{2}t^2 + C = -16t^2 + C$

LET t_g = TIME STONE HITS GROUND

THE HEIGHT OF CLIFF $= s(0) - s(t_g)$

INITIAL HEIGHT FINAL HEIGHT

FIND t_g: $v(t_g) = -120 \Rightarrow -32t_g = -120$

$$t_g = \frac{120}{32} = \frac{15}{4}$$

$s(t) = -32 \cdot \frac{1}{2} t^2 + C = -16t^2 + C$

THE HEIGHT OF CLIFF $= s(0) - s(t_g)$

$$= -16(0)^2 + C - \left[-16\left(\frac{15}{4}\right)^2 + C\right]$$

$$= C + 16\left(\frac{15}{4}\right)^2 - C$$

$$= 15^2 = 225 \text{ ft}$$

65. A company estimates that the marginal cost (in dollars per item) of producing x items is $1.92 - 0.002x$. If the cost of producing one item is \$562, find the cost of producing 100 items.