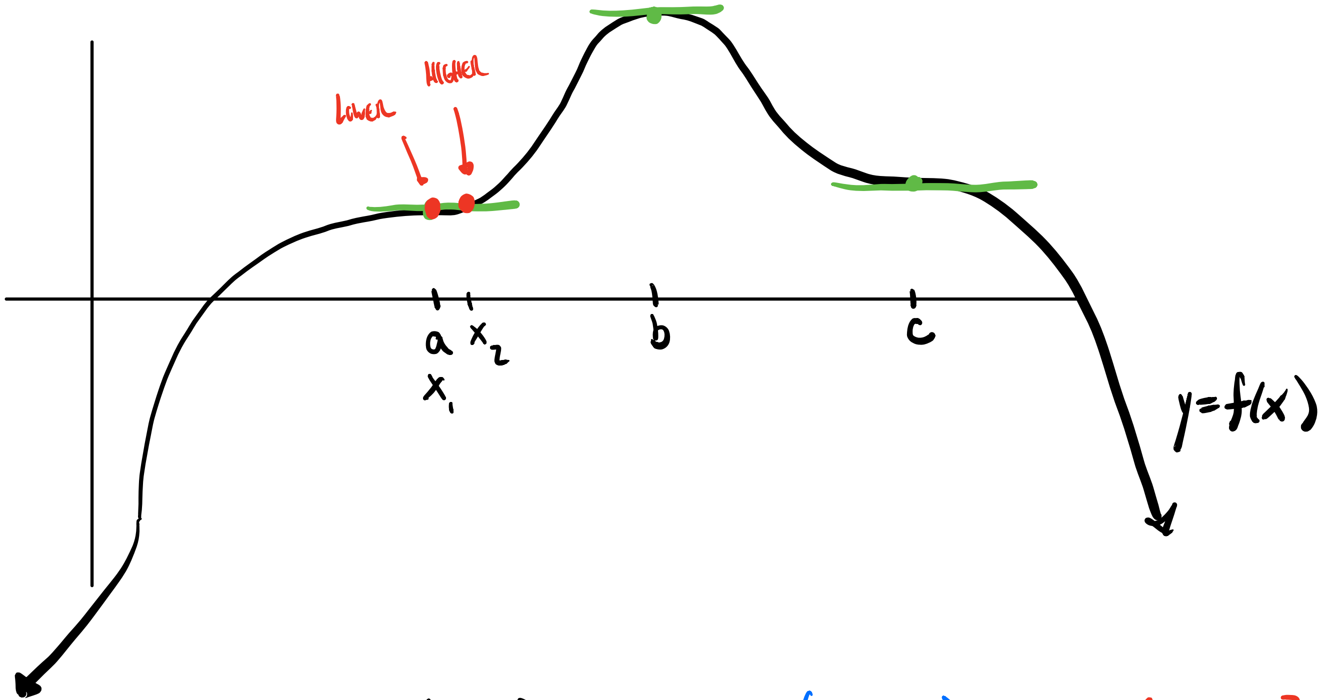

INCREASING : $(-\infty, a) \cup (a, b)$ OR $(-\infty, b)$ OR $(-\infty, b]$

DECREASING : $(b, c) \cup (c, \infty)$ OR (b, ∞) OR $[b, \infty)$

f INCR. AT a IF $f'(a) > 0$, SIM. DECR.

Def: f IS INCR. ON I IF FOR ANY $x_1, x_2 \in I$ WITH $x_1 < x_2$, WE HAVE $f(x_1) < f(x_2)$

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§4.5 The Substitution Rule

The chain rule: used when taking the derivative of the composition of two or more functions.

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\,g'(x)$$

"Anti-chain-rule"

$$\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$$

Inner function

Derivative of inner function

ex. $\int 2x \sin(x^2)\,dx$

Inner function x^2

1. Introduce new variable for inner func.

$u = x^2$

u is a differentiable function of x.

Translate from x to u

2. Differentials

$\frac{du}{dx} = 2x \rightarrow du = 2x\,dx$

$$\int \sin(x^2)\,2x\,dx \rightarrow \int \sin(u)\,du$$

4. Translate

5. INTEGRATE

$= -\cos(u) + C$ (INTEGRATION NOW OVER)

6. BACK - SUBSTITUTE

$\rightsquigarrow \boxed{-\cos(x^2) + C}$

CHECK: $\frac{d}{dx}\left[-\cos(x^2) + C\right]$

$= \sin(x^2) \cdot 2x$ ✓

U - SUBSTITUTION:

$$\int f'(g(x))\, g'(x)\, dx$$

LET $u = g(x)$

$du = g'(x)\, dx$

$$\rightsquigarrow \int f'(u)\, du = f(u) + C \rightsquigarrow f(g(x)) + C$$

ex. $\int \sin^3 x \cos x \, dx$

$\hookrightarrow \int (\sin x)^3 \cos x \, dx$

TIP: LOOK FOR FUNCTION COMPOSITION $f(g(x))$

LET $u =$ INNER FUNCTION.

FIGURE OUT AS YOU GO.

$u = \sin x$

$du = \cos x \, dx$

$\frac{du}{dx} = \cos x$

$\rightsquigarrow \int u^3\,du = \frac{1}{4}u^4 + C$

$\rightsquigarrow \boxed{\frac{1}{4}\sin^4 x + C}$

CHECK: $\frac{d}{dx}\left[\frac{1}{4}\sin^4 x + C\right]$

$\frac{1}{\cancel{4}}\cdot\cancel{4}\sin^3 x\cos x$ ✓

ex. $\int \frac{x}{\sqrt{1-4x^2}}\,dx$

THERE ARE FUNCTIONS INSIDE FUNCTIONS

LET $u = 1-4x^2$

$du = -8x\,dx$

$-\frac{1}{8}\,du = x\,dx$

$\rightsquigarrow \int \frac{1}{\sqrt{u}}\,\underbrace{x\,dx}$

ALMOST du :(

CONSTANT MULTIPLE OF du

NO-NO: x's & u's TOGETHER!

WE STUDY SINGLE VARIABLE CALCULUS!

$\rightsquigarrow \int \frac{1}{\sqrt{u}}\cdot -\frac{1}{8}\,du = -\frac{1}{8}\int u^{-1/2}\,du$

$= -\frac{1}{8}\left(2u^{1/2}\right) + C = -\frac{1}{4}u^{1/2} + C$

$\rightsquigarrow \boxed{-\frac{1}{4}\sqrt{1-4x^2} + C}$

U-SUBSTITUTION WITH DEFINITE INTEGRAL:

$$\int_a^b f'(g(x))\,g'(x)\,dx$$

LET $u = g(x)$

$du = g'(x)\,dx$

$$\rightsquigarrow \int_{g(a)}^{g(b)} f'(u)\,du$$

$a \leq x \leq b$

$g(a) \leq g(x) \leq g(b)$

$g(a) \leq u \leq g(b)$

$$= f(g(b)) - f(g(a)).$$

ex. $$\int_0^1 (3t-1)^{50}\,dt$$

LET $u = 3t-1$

$du = 3\,dt$

$\downarrow$ SOLVE FOR A TERM IN INTEGRAND

$\frac{1}{3}\,du = dt$

$$\rightsquigarrow \frac{1}{3}\int_{-1}^{2} u^{50}\,du$$

$t=1 \dashrightarrow u=2$

$u = 3t-1$

$t=0 \dashrightarrow u=-1$

$$= \frac{1}{3}\left(\frac{1}{51}u^{51}\right)\Bigg|_{-1}^{2} = \boxed{\frac{1}{153}\left(2^{51} - (-1)^{51}\right)}$$

Alt:

$$\frac{1}{153}(3t-1)^{51}\Big|_0^1 = \frac{1}{153}\left[(3(1)-1)^{51} - (3(0)-1)^{51}\right]$$

$$= \frac{1}{153}\left[(2)^{51} - (-1)^{51}\right]$$

ex.

$$\int_0^{\pi/2} \sin^6(x)\cos^5(x)\,dx = \int_0^{\pi/2} \sin^6(x)\cos^4(x)\cos(x)\,dx$$

$$= \int_0^{\pi/2} \sin^6 x\left(\cos^2 x\right)^2 \cos x\,dx$$

$$= \int_0^{\pi/2} \sin^6 x\left(1-\sin^2 x\right)^2 \cos x\,dx$$

Let $u = \sin x$

$du = \cos x\,dx$

$$\rightsquigarrow \int_0^1 u^6(1-u^2)\,du = \int_0^1 u^6 - u^8\,du$$

$$= \left[\frac{1}{7}u^7 - \frac{1}{9}u^9\right]_0^1 = \frac{1}{7} - \frac{1}{9} = \boxed{\frac{2}{63}}$$

$$\int (x+3)\sqrt{x-1}\,dx$$

LET $u = x - 1$

$\quad +4 \qquad +4$

$u + 4 = x + 3$ *

$du = dx$

$$\sim \int (u+4)\sqrt{u}\,du = \int u^{3/2} + 4u^{1/2}\,du$$

$$\frac{2}{5}u^{5/2} + \frac{8}{3}u^{3/2} + C$$

$$\sim \boxed{\frac{2}{5}(x-1)^{5/2} + \frac{8}{3}(x-1)^{3/2} + C}$$