10/02/2020 2.8 Related Rates

Keyword: "rate of change of y with respect to x"

$\rightarrow \frac{dy}{dx}$.

Example 1. Air is being pumped into a spherical balloon. Volume increases at a "rate" of 100 cm^3/sec. How "fast" is the radius of the balloon increasing when the diameter is 50 cm?

* let V : volume of the balloon.

r : radius of the balloon.

formula of sphere volume : $V(t) = \frac{4}{3}\pi r^3(t)$

we want to know $\frac{dr}{dt}$ when $r = 50/2 = 25$ cm.

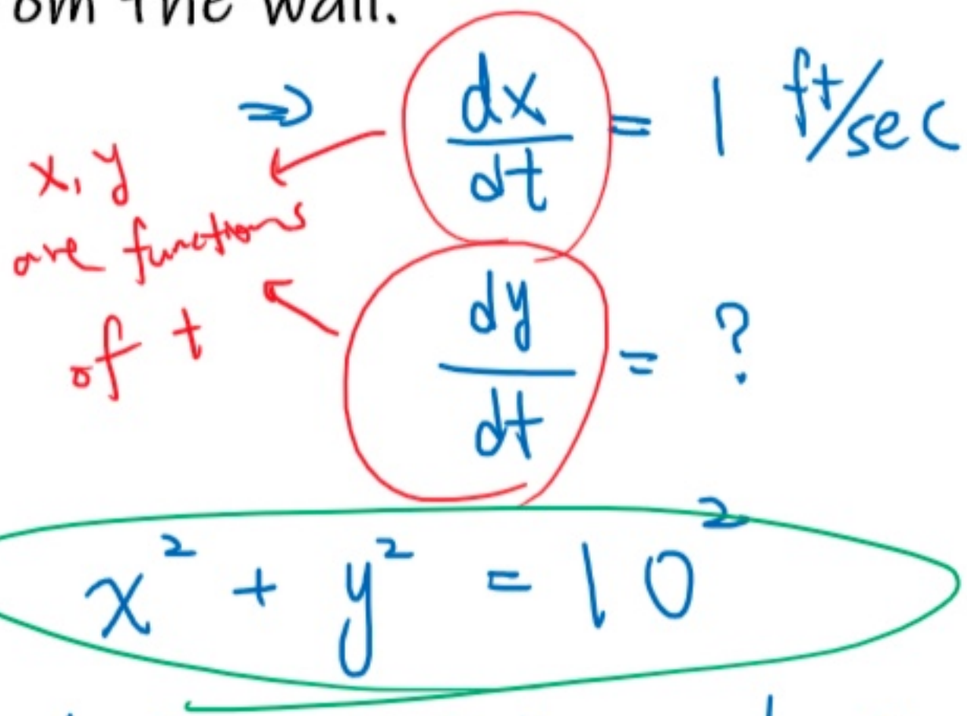
$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

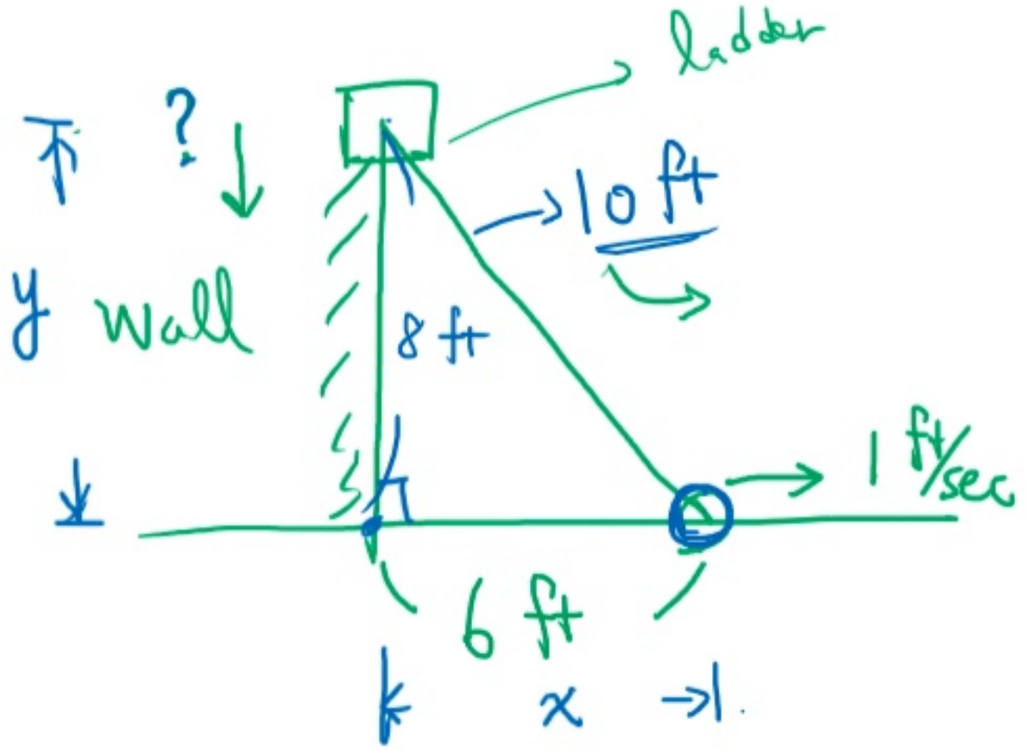
$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3 r^2 \frac{dr}{dt}.$$

→ Chain Rule.

$$100 = 4\pi (25)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{25\pi}$$

Example 2. (sliding ladder problem) Bottom of the ladder slides away from the wall at a rate of 1 ft/sec. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall.





$$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt}(100)$$

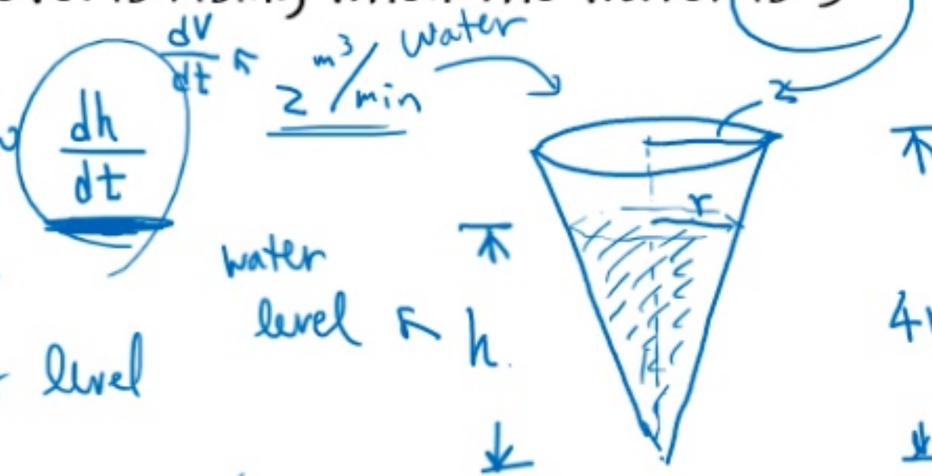
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\Rightarrow 2(6)\cdot 1 + 2\cdot 8\cdot \frac{dy}{dt} = 0$$

$$\Rightarrow 12 + 16\frac{dy}{dt} = 0.$$

$$\frac{dy}{dt} = -\frac{12}{16} = \boxed{-\frac{3}{4}} \text{ ft/sec}$$

Example 3. (Water tank problem) A water tank has cone shape with base radius = 2 meter and height 4 meter. Water is being pumped into the tank at a rate of $2m^3/min$. Find the rate at which the water level is rising when the water is 3 meter deep.



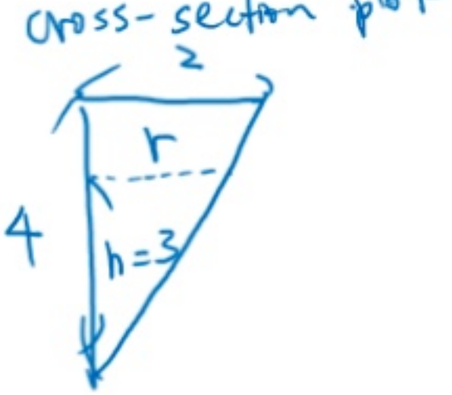
We want to know $\frac{dh}{dt}$

let V : volume of water

h : height of water level

$\frac{dV}{dt}$: $2 \frac{m^3}{min}$ ✓

Formula of cone volume : $V = \frac{1}{3}\pi r^2 h$ → depends on t



From the similar triangle

$$\frac{h}{r} = \frac{4}{2} = \frac{2}{1} \Rightarrow h = 2r \Rightarrow r = \frac{h}{2}$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{1}{12}\pi h^3\right) = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$\Rightarrow \quad 2 = \frac{1}{4}\pi \cdot 3^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{8}{9\pi} \frac{m}{min}$$