Method for finding extrema: Closed Interval Method

Goal: find the absolute maximum/minimum of f on $[a, b]$

STEPS:

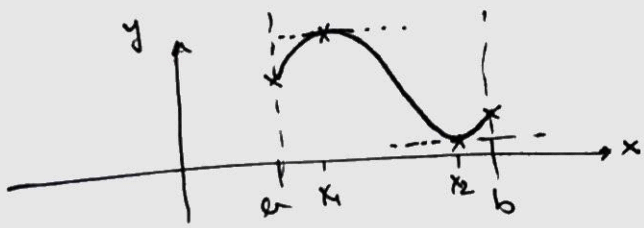
1: Find $f'(x) = 0$. $x \in (a, b)$. ← critical numbers

2: Check endpoints $f(a)$, $f(b)$.

3. The largest value of STEP 1 & 2 is the abs. max.

The smallest value of STEP 1 & 2 is the abs. min.

Schematic.



Compare $f(x_1)$, $f(x_2)$, $f(a)$, $f(b)$ to find the abs. max/min

Notice: This method can only be used when f is continuous on $[a, b]$

Example 1. Find the abs. max. and abs. min of f on $[-\frac{1}{2}, 4]$.

CHECK. f is cont. ✓ can use the method.

$f(x) = x^3 - 3x^2 + 1$, $-\frac{1}{2} \le x \le 4$.

① Find $f'(x) = 0$. $f'(x) = 3x^2 - 6x = 3x(x-2)$.

$f'(x) = 0$ if $\underline{x = 0}$ or $\underline{x = 2}$.

$f(0) = 1$, $f(2) = 8 - 3 \cdot 4 + 1 = \boxed{-3}$

② Check endpoints: $f(-\frac{1}{2}) = (\frac{-1}{2})^3 - 3(\frac{-1}{2})^2 + 1 = \frac{-1}{8} - \frac{3}{4} + 1 = \frac{1}{8}$

$f(4) = 4^3 - 3 \cdot 4^2 + 1 = 64 - 48 + 1 = \boxed{17}$

③ The abs. max is 17

abs. min is -3

Exercise # 55 $f(t) = 2\cos t + \sin(2t)$ $[0, \frac{\pi}{2}]$ → continuous.

Find abs. max/min.

① $f'(t) = -2\sin t + 2\cos(2t)$

$= -2\sin t + 2(\cos t \cancel{\cos t} - \sin t \sin t)$. $\cos^2 t = 1 - \sin^2 t$.

$= -2\sin t + 2(\cos^2 t - \sin^2 t)$

$= -2\sin t + 2(1 - 2\sin^2 t)$.

$f'(t) = 0 \Rightarrow -2\sin t - 4\sin^2 t + 2 = 0$.

$\div 2$

$\Rightarrow 2\sin^2 t + \sin t - 1 = 0$.

$\Rightarrow (2\sin t - 1)(\sin t + 1) = 0$.

$\Rightarrow 2\sin t - 1 = 0$ or $\sin t = -1$.

$\Rightarrow \begin{cases} \sin t = \frac{1}{2}, & t = \frac{\pi}{6} \\ \sin t = -1, & t = \frac{3\pi}{2} \end{cases}$

✓ $f(\frac{\pi}{6}) = 2\cos\frac{\pi}{6} + \sin(\frac{\pi}{3}) = 2\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \boxed{\frac{3\sqrt{3}}{2}}$

$t = \frac{3\pi}{2}$ → not in $[0, \frac{\pi}{2}]$. X.

② Check endpoints.

$f(0) = 2\cos 0 + \sin 0 = 2$.

$f(\frac{\pi}{2}) = 2\cos\frac{\pi}{2} + \sin(\pi) = \boxed{0}$

③ The abs. max is $\frac{3\sqrt{3}}{2}$

The abs. min is 0