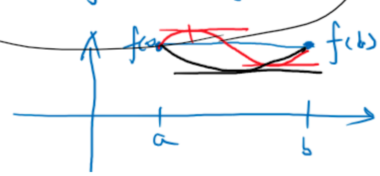
§3.2 The Mean Value Theorem

① Rolle's Theorem: If f is ← special case

(1) continuous on $[a, b]$, &

(2) differentiable on (a, b) &

(3) $f(a) = f(b)$.



Then there exists a number C such that $f'(c) = 0$.

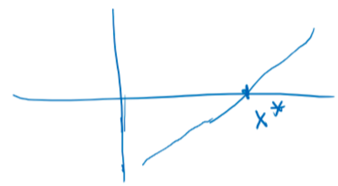
Example 1. Show that $x^3 + x - 1 = 0$ has exactly one root.

Let $f(x) = x^3 + x - 1$. x^* is a root $\Rightarrow f(x^*) = 0$.

proof by contradiction

↓

$f'(x) = 3x^2 + 1 > 0$.



Assumption!

PROOF: Suppose we have 2 roots, a & b,

this implies that $f(a) = 0$ & $f(b) = 0$.

but, from Rolle's thm, we know that there exists a number C such that $f'(c) = 0$. and $f'(x) > 0$, there is a "contradiction".

Therefore, there is only one root! □

The Mean Value Theorem.

If f is ① continuous on $[a, b]$

② differentiable on (a, b)

Then there is a number c such that

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$