§8.1 Arc Length

9–20 Find the exact length of the curve.

9. $y = 1 + 6x^{3/2}, \quad 0 \leqslant x \leqslant 1$

10. $36y^2 = (x^2 - 4)^3, \quad 2 \leqslant x \leqslant 3, \quad y \geqslant 0$

11. $y = \dfrac{x^3}{3} + \dfrac{1}{4x}, \quad 1 \leqslant x \leqslant 2$

12. $x = \dfrac{y^4}{8} + \dfrac{1}{4y^2}, \quad 1 \leqslant y \leqslant 2$

13. $x = \frac{1}{3}\sqrt{y}\,(y - 3), \quad 1 \leqslant y \leqslant 9$

14. $y = \ln(\cos x), \quad 0 \leqslant x \leqslant \pi/3$

15. $y = \ln(\sec x), \quad 0 \leqslant x \leqslant \pi/4$

16. $y = 3 + \frac{1}{2}\cosh 2x, \quad 0 \leqslant x \leqslant 1$

17. $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \quad 1 \leqslant x \leqslant 2$

18. $y = \sqrt{x - x^2} + \sin^{-1}\left(\sqrt{x}\right)$

19. $y = \ln(1 - x^2), \quad 0 \leqslant x \leqslant \frac{1}{2}$

20. $y = 1 - e^{-x}, \quad 0 \leqslant x \leqslant 2$

§8.2 Surface Areas

7–14 Find the exact area of the surface obtained by rotating the curve about the x-axis.

7. $y = x^3, \ 0 \leqslant x \leqslant 2$

8. $y = \sqrt{5 - x}, \ 3 \leqslant x \leqslant 5$

9. $y^2 = x + 1, \ 0 \leqslant x \leqslant 3$

10. $y = \sqrt{1 + e^x}, \ 0 \leqslant x \leqslant 1$

11. $y = \cos\left(\frac{1}{2}x\right), \ 0 \leqslant x \leqslant \pi$

12. $y = \dfrac{x^3}{6} + \dfrac{1}{2x}, \ \frac{1}{2} \leqslant x \leqslant 1$

13. $x = \frac{1}{3}(y^2 + 2)^{3/2}, \ 1 \leqslant y \leqslant 2$

14. $x = 1 + 2y^2, \ 1 \leqslant y \leqslant 2$

15–18 The given curve is rotated about the y-axis. Find the area of the resulting surface.

15. $y = \frac{1}{3}x^{3/2}, \ 0 \leqslant x \leqslant 12$

16. $x^{2/3} + y^{2/3} = 1, \ 0 \leqslant y \leqslant 1$

17. $x = \sqrt{a^2 - y^2}, \ 0 \leqslant y \leqslant a/2$

18. $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \ 1 \leqslant x \leqslant 2$

§8.3 Hydrostatic Force

1. An aquarium 5 ft long, 2 ft wide, and 3 ft deep is full of water. Find (a) the hydrostatic pressure on the bottom of the aquarium, (b) the hydrostatic force on the bottom, and (c) the hydrostatic force on one end of the aquarium.

2. A tank is 8 m long, 4 m wide, 2 m high, and contains kerosene with density 820 kg/m^3 to a depth of 1.5 m. Find (a) the hydrostatic pressure on the bottom of the tank, (b) the hydrostatic force on the bottom, and (c) the hydrostatic force on one end of the tank.

3–11 A vertical plate is submerged (or partially submerged) in water and has the indicated shape. Explain how to approximate the hydrostatic force against one side of the plate by a Riemann sum. Then express the force as an integral and evaluate it.

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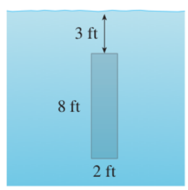

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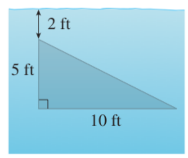

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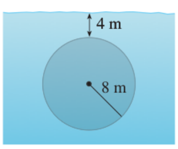

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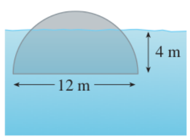

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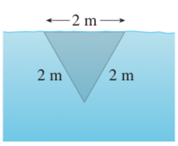

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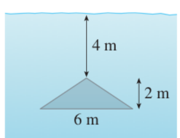

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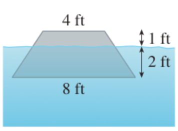

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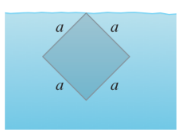

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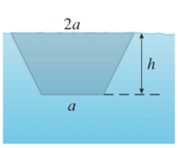


§10.1 Parametric Eq's

19–22 Describe the motion of a particle with position (x, y) as t varies in the given interval.

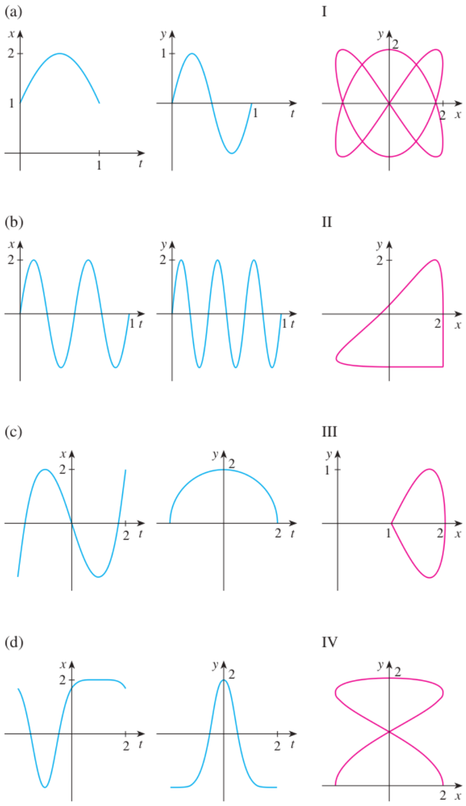
19. $x = 5 + 2\cos \pi t, \quad y = 3 + 2\sin \pi t, \quad 1 \leqslant t \leqslant 2$

20. $x = 2 + \sin t, \quad y = 1 + 3\cos t, \quad \pi/2 \leqslant t \leqslant 2\pi$

21. $x = 5\sin t, \quad y = 2\cos t, \quad -\pi \leqslant t \leqslant 5\pi$

22. $x = \sin t, \quad y = \cos^2 t, \quad -2\pi \leqslant t \leqslant 2\pi$

24. Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)–(d) with the parametric curves labeled I–IV. Give reasons for your choices.



33. Find parametric equations for the path of a particle that moves along the circle $x^2 + (y - 1)^2 = 4$ in the manner described.

(a) Once around clockwise, starting at (2, 1)
(b) Three times around counterclockwise, starting at (2, 1)
(c) Halfway around counterclockwise, starting at (0, 3)

§10.2 CALCULUS WITH PARAMETRIC EQ's

3–6 Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

3. $x = t^3 + 1, \quad y = t^4 + t; \quad t = -1$

4. $x = \sqrt{t}, \quad y = t^2 - 2t; \quad t = 4$

5. $x = t\cos t, \quad y = t\sin t; \quad t = \pi$

6. $x = e^t \sin \pi t, \quad y = e^{2t}; \quad t = 0$

17–20 Find the points on the curve where the tangent is horizontal or vertical. If you have a graphing device, graph the curve to check your work.

17. $x = t^3 - 3t, \quad y = t^2 - 3$

18. $x = t^3 - 3t, \quad y = t^3 - 3t^2$

19. $x = \cos\theta, \quad y = \cos 3\theta$

20. $x = e^{\sin\theta}, \quad y = e^{\cos\theta}$

27. (a) Find the slope of the tangent line to the trochoid $x = r\theta - d\sin\theta$, $y = r - d\cos\theta$ in terms of θ. (See Exercise 10.1.40.)
(b) Show that if $d < r$, then the trochoid does not have a vertical tangent.

28. (a) Find the slope of the tangent to the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ in terms of θ. (Astroids are explored in the Laboratory Project on page 689.)
(b) At what points is the tangent horizontal or vertical?
(c) At what points does the tangent have slope 1 or -1?

29. At what point(s) on the curve $x = 3t^2 + 1$, $y = t^3 - 1$ does the tangent line have slope $\frac{1}{2}$?

30. Find equations of the tangents to the curve $x = 3t^2 + 1$, $y = 2t^3 + 1$ that pass through the point (4, 3).

37–40 Set up an integral that represents the length of the curve. Then use your calculator to find the length correct to four decimal places.

37. $x = t + e^{-t}, \quad y = t - e^{-t}, \quad 0 \leqslant t \leqslant 2$

38. $x = t^2 - t, \quad y = t^4, \quad 1 \leqslant t \leqslant 4$

39. $x = t - 2\sin t, \quad y = 1 - 2\cos t, \quad 0 \leqslant t \leqslant 4\pi$

40. $x = t + \sqrt{t}, \quad y = t - \sqrt{t}, \quad 0 \leqslant t \leqslant 1$

41–44 Find the exact length of the curve.

41. $x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leqslant t \leqslant 1$

42. $x = e^t - t, \quad y = 4e^{t/2}, \quad 0 \leqslant t \leqslant 2$

43. $x = t\sin t, \quad y = t\cos t, \quad 0 \leqslant t \leqslant 1$

44. $x = 3\cos t - \cos 3t, \quad y = 3\sin t - \sin 3t, \quad 0 \leqslant t \leqslant \pi$

§10.3 Polar Coord

5–6 The Cartesian coordinates of a point are given.
(i) Find polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leqslant \theta < 2\pi$.
(ii) Find polar coordinates (r, θ) of the point, where $r < 0$ and $0 \leqslant \theta < 2\pi$.

5. (a) $(-4, 4)$ (b) $(3, 3\sqrt{3})$

6. (a) $(\sqrt{3}, -1)$ (b) $(-6, 0)$

7–12 Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions.

7. $r \geqslant 1$

8. $0 \leqslant r < 2, \quad \pi \leqslant \theta \leqslant 3\pi/2$

9. $r \geqslant 0, \quad \pi/4 \leqslant \theta \leqslant 3\pi/4$

10. $1 \leqslant r \leqslant 3, \quad \pi/6 < \theta < 5\pi/6$

11. $2 < r < 3, \quad 5\pi/3 \leqslant \theta \leqslant 7\pi/3$

12. $r \geqslant 1, \quad \pi \leqslant \theta \leqslant 2\pi$

29–46 Sketch the curve with the given polar equation by first sketching the graph of r as a function of θ in Cartesian coordinates.

29. $r = -2\sin\theta$ **30.** $r = 1 - \cos\theta$

31. $r = 2(1 + \cos\theta)$ **32.** $r = 1 + 2\cos\theta$

33. $r = \theta, \ \theta \geqslant 0$

34. $r = \theta^2, \ -2\pi \leqslant \theta \leqslant 2\pi$

35. $r = 3\cos 3\theta$ **36.** $r = -\sin 5\theta$

37. $r = 2\cos 4\theta$ **38.** $r = 2\sin 6\theta$

39. $r = 1 + 3\cos\theta$ **40.** $r = 1 + 5\sin\theta$

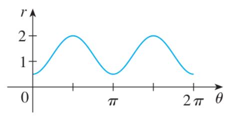
41. $r^2 = 9\sin 2\theta$ **42.** $r^2 = \cos 4\theta$

43. $r = 2 + \sin 3\theta$ **44.** $r^2\theta = 1$

45. $r = \sin(\theta/2)$ **46.** $r = \cos(\theta/3)$

47–48 The figure shows a graph of r as a function of θ in Cartesian coordinates. Use it to sketch the corresponding polar curve.

47.



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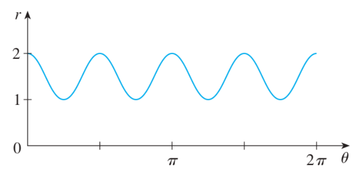

§10.4 Calculus with Polar Coord.

17–21 Find the area of the region enclosed by one loop of the curve.

17. $r = 4\cos 3\theta$ **18.** $r^2 = 4\cos 2\theta$

19. $r = \sin 4\theta$ **20.** $r = 2\sin 5\theta$

21. $r = 1 + 2\sin\theta$ (inner loop)

22. Find the area enclosed by the loop of the **strophoid** $r = 2\cos\theta - \sec\theta$.

23–28 Find the area of the region that lies inside the first curve and outside the second curve.

23. $r = 4\sin\theta, \quad r = 2$

24. $r = 1 - \sin\theta, \quad r = 1$

25. $r^2 = 8\cos 2\theta, \quad r = 2$

26. $r = 1 + \cos\theta, \quad r = 2 - \cos\theta$

27. $r = 3\cos\theta, \quad r = 1 + \cos\theta$

28. $r = 3\sin\theta, \quad r = 2 - \sin\theta$

29–34 Find the area of the region that lies inside both curves.

29. $r = 3\sin\theta, \quad r = 3\cos\theta$

30. $r = 1 + \cos\theta, \quad r = 1 - \cos\theta$

31. $r = \sin 2\theta, \quad r = \cos 2\theta$

32. $r = 3 + 2\cos\theta, \quad r = 3 + 2\sin\theta$

33. $r^2 = 2\sin 2\theta, \quad r = 1$

34. $r = a\sin\theta, \quad r = b\cos\theta, \quad a > 0, \ b > 0$

55–60 Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

55. $r = 2\cos\theta, \quad \theta = \pi/3$ **56.** $r = 2 + \sin 3\theta, \quad \theta = \pi/4$

57. $r = 1/\theta, \quad \theta = \pi$ **58.** $r = \cos(\theta/3), \quad \theta = \pi$

59. $r = \cos 2\theta, \quad \theta = \pi/4$ **60.** $r = 1 + 2\cos\theta, \quad \theta = \pi/3$

61–64 Find the points on the given curve where the tangent line is horizontal or vertical.

61. $r = 3\cos\theta$ **62.** $r = 1 - \sin\theta$

63. $r = 1 + \cos\theta$ **64.** $r = e^\theta$