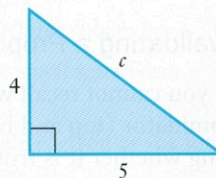


9. The longest side of a right triangle is called the
- Pythagorean theorem.
 - hypotenuse.
 - slant.
10. Consider the right triangle below, with the two known side lengths indicated and the unknown side length c . Which statement is true?
- $c = 6$
 - $c^2 = 41$
 - $c = 41$



Exercises 1D

REVIEW QUESTIONS

- Summarize the differences between deductive and inductive arguments. Give an example of each type.
- Briefly explain the idea of strength and how it applies to inductive arguments. Can an inductive argument prove its conclusion true? Can an inductive argument be valid? Can it be sound?
- Briefly explain the ideas of validity and soundness and how they apply to deductive arguments. Can a valid deductive argument be unsound? Can a sound deductive argument be invalid? Explain.
- Describe the procedure used to test the validity of a deductive argument with a Venn diagram.
- Create your own example of each of the four basic conditional arguments. Then explain why your argument is valid or invalid.
- What is a chain of conditionals? Give an example of a valid argument made from such a chain.
- Can inductive logic be used to prove a mathematical theorem? Explain.
- How can inductive testing of a mathematical rule be useful? Give an example.

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

- Based on the testimonials of dozens of people who have lost weight following my diet, I will prove to you that my diet works for everyone.
- The many examples of people whose cancer went away following chemotherapy make a strong case for the idea that chemotherapy can cure cancer.
- Through the logic of deduction, I will show you that if you accept the truth of my premises, you must also accept the truth of my conclusion.
- You can see that my argument is valid, and you must therefore accept the truth of my conclusion.
- If you use logic, then your life will be organized. Therefore, if your life is organized, you must be using logic.
- Even before Fermat's Last Theorem was proved deductively, mathematicians were sure it was true.

BASIC SKILLS & CONCEPTS

15–22: **Everyday Logic.** Explain whether the following arguments are deductive or inductive.

- I have never found mail in my mailbox on a Sunday. The Postal Service must not have Sunday deliveries.
- Because of a budget cutback, postal workers will no longer work on Saturdays. Therefore, I will not expect Saturday deliveries in the future.
- Eminem's first five CDs were outstanding. His next CD is bound to be good, so I will buy it without even listening to it.
- If I eat spicy food before noon, then I get indigestion in the afternoon. Whenever I get indigestion, I have no appetite for the next six hours. Therefore, if I eat spicy food before noon, then I cannot eat dinner.
- January is windier than July. The wind must blow more often in the winter than in the summer.
- If a natural number is divisible by 2, then it is even. The number 24 is divisible by 2. Therefore, 24 is even.
- The Flanagans are having their fourth child. Their other three children are musically gifted, so the youngest child is bound to be musically gifted as well.
- If I have breakfast at Sid's Café, then I can park free; but then I walk to work, which makes me late, which makes my boss mad. So if I have breakfast at Sid's Café, I save money and make my boss mad.

23–28: Analyzing Inductive Arguments. Determine the truth of the premises of the following arguments. Then assess the strength of the argument and discuss the truth of the conclusion.

23. Premise: $2 + 3 = 5$
 Premise: $5 + 4 = 9$
 Premise: $7 + 6 = 13$
 Conclusion: The sum of an even integer and an odd integer is an odd integer.
24. Premise: If I pay more for a pair of running shoes, they last longer.
 Premise: If I pay more for an automobile, it requires fewer repairs.
 Conclusion: Quality goes with high prices.
25. Premise: Trout and bass swim and they are fish.
 Premise: Sharks and marlin swim and they are fish.
 Premise: Tuna and salmon swim and they are fish.
 Conclusion: Whales swim and they are fish.
26. Premise: Apes and baboons have hair and they are mammals.
 Premise: Mice and rats have hair and they are mammals.
 Premise: Tigers and lions have hair and they are mammals.
 Conclusion: Animals with hair are mammals.
27. Premise: $(-6) \times (-4) = 24$
 Premise: $(-2) \times (-1) = 2$
 Premise: $(-27) \times (-3) = 81$
 Conclusion: Whenever we multiply two negative numbers, the result is a positive number.
28. Premise: Bach, Buxtehude, Beethoven, Brahms, Berlioz, and Britten are great composers.
 Conclusion: Composers with names that begin with B are great.

29–36: Analyzing Deductive Arguments. Consider the following arguments.

a. If necessary, rephrase the first premise so that it has the form *all S are P*.

b. Draw a Venn diagram to determine whether the argument is valid.

c. Discuss the truth of the premises and state whether the argument is sound.

29. Premise: All European countries use the euro as currency.
 Premise: Great Britain is a European country.
 Conclusion: Great Britain uses the euro as currency.
30. Premise: All dairy products contain protein.
 Premise: Soybeans contain protein.
 Conclusion: Soybeans are dairy products.
31. Premise: No states west of the Mississippi River are in the eastern time zone.
 Premise: Utah is west of the Mississippi River.
 Conclusion: Utah is not in the eastern time zone.
32. Premise: All U.S. presidents have been men.
 Premise: George Washington was a man.
 Conclusion: George Washington was a U.S. president.
33. Premise: All Best Actor Academy Award winners have been men.
 Premise: Sean Penn is a man.
 Conclusion: Sean Penn won a Best Actor Academy Award.
34. Premise: All fruit is fat-free.
 Premise: Avocados are fruit.
 Conclusion: Avocados are fat-free.
35. Premise: All opera singers can whistle a Mozart tune.
 Premise: Pavarotti was an opera singer.
 Conclusion: Pavarotti could whistle a Mozart tune.
36. Premise: No country is an island.
 Premise: Iceland is a country.
 Conclusion: Iceland is not an island.

37–44: Deductive Arguments with Conditional Propositions. Consider the following arguments.

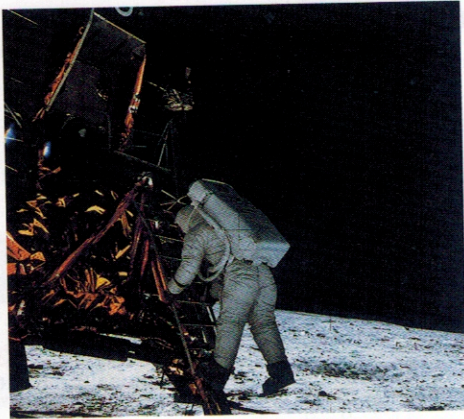
a. If necessary, rephrase the first premise so that it has the form *if p, then q*.

b. Identify the type of argument and determine its validity with a Venn diagram.

c. Discuss the truth of the premises and state whether the argument is sound.

37. Premise: If an animal is a dog, then it is a mammal.
 Premise: Setters are dogs.
 Conclusion: Setters are mammals.
38. Premise: If it is a bird, then its young are hatched from eggs.
 Premise: Condors are birds.
 Conclusion: Condor chicks are hatched from eggs.
39. Premise: If you live in Boston, you live in Massachusetts.
 Premise: Amanda does not live in Boston.
 Conclusion: Amanda does not live in Massachusetts.
40. Premise: If you live in Boston, you live in Massachusetts.
 Premise: Bruno lives in Massachusetts.
 Conclusion: Bruno lives in Boston.
41. Premise: If a figure is a triangle, then it has three sides.
 Premise: Squares have four sides.
 Conclusion: Squares are not triangles.
42. Premise: It's necessary for nurses to know CPR.
 Premise: Tom is a nurse.
 Conclusion: Tom knows CPR.
43. Premise: Novels written in the 19th century were not written on a word processor.
 Premise: Jake finished his first novel last year.
 Conclusion: Jake's first novel was written on a word processor.

44. Premise: If we can put a man on the Moon, we can build a computer operating system that works.
 Premise: We can build a computer operating system that works.
 Conclusion: We can put a man on the Moon.



45–48: **Chains of Conditionals.** Write the given argument as a chain of conditional propositions that have the form *if p, then q*. Then determine the validity of the entire argument.

45. Premise: If a natural number is divisible by 8, then it is divisible by 4.
 Premise: If a natural number is divisible by 4, then it is divisible by 2.
 Conclusion: If a natural number is divisible by 8, then it is divisible by 2.
46. Premise: If a creature is a reptile, then it is an animal.
 Premise: If a creature is an animal, then it is alive.
 Conclusion: If a creature is alive, then it is a reptile.
47. Premise: If taxes are increased, then taxpayers will have less disposable income.
 Premise: With less disposable income, spending will decrease and the economy will slow down.
 Conclusion: A tax increase will slow down the economy.
48. Premise: If taxes are cut, the U.S. government will have less revenue.
 Premise: If there is less revenue, then the deficit will be larger.
 Conclusion: Tax cuts will lead to a larger deficit.

49–51: **Testing Mathematical Rules.** Test the following rules with several different sets of numbers. If possible, try to find a counterexample (a set of numbers for which the rule is not true). State whether you think the rule is true.

49. Is it true for all real numbers a and b that $a + b = b + a$?
50. Is it true for all nonzero real numbers a , b , and c that

$$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}?$$

51. It is true for all positive real numbers a and b that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$?

52. It is true for all positive integers n that

$$1 + 2 + 3 + \cdots + n = \frac{n \times (n + 1)}{2}?$$

FURTHER APPLICATIONS

53–57: **Validity and Soundness.** State whether it is possible for a deductive argument to have the following properties. If so, make a simple three-proposition argument that demonstrates your conclusion.

53. Valid and sound
 54. Not valid and sound
 55. Valid and not sound
 56. Valid with false premises and a true conclusion
 57. Not valid with true premises and a true conclusion

58–61: **Make Your Own Argument.** Create simple three-proposition arguments that have the following forms.

58. Affirming the hypothesis
 59. Affirming the conclusion
 60. Denying the hypothesis
 61. Denying the conclusion

62. **The Goldbach Conjecture.** Recall that a prime number is a natural number whose only factors are itself and 1 (for example, 2, 3, 5, 7, 11, ...). The Goldbach conjecture, posed in 1742, claims that every even number greater than 2 can be expressed as the sum of two primes. For example, $4 = 2 + 2$, $6 = 3 + 3$, and $8 = 5 + 3$. A *deductive* proof of this conjecture has never been found. Test the conjecture for at least 10 even numbers and present an *inductive* argument for its truth. Do you think the conjecture is true? Why or why not?

63–65: **Conditionals in the Literature.** Consider the following propositions and answer the questions that follow.

63. “If Toyota takes market share from Ford by manufacturing hybrids while Ford is still making SUVs, Ford is punished by the market.” —Thomas Friedman, *Hot, Flat, and Crowded*
- a. What is the logical conclusion (if any) in the event that Toyota takes market share from Ford by manufacturing hybrids while Ford is still making SUVs?
- b. What is the logical conclusion (if any) in the event that Ford is not punished by the market?
- c. Can you draw any conclusion about what happens if Toyota makes SUVs and Ford makes hybrids?
64. “If floods have not occurred in the immediate past, people who live on the flood plains are far less likely to purchase flood insurance.” —Cass Sunstein, *Worst-Case Scenarios*
- a. What is the logical conclusion (if any) in the event that floods have not occurred in the immediate past?
- b. What is the logical conclusion (if any) in the event that people who live on the flood plains purchase flood insurance?
- c. Can you draw any conclusion about what happens if floods have occurred in the immediate past?