

## QUICK QUIZ

Choose the best answer to each of the following questions.  
Explain your reasoning with one or more complete sentences.

- What does the word *per* mean?
  - divided by
  - multiplied by
  - in addition to
- Which of the following represents 4 square miles?
  - a line of small squares that is 4 miles long
  - a square 2 miles on a side
  - a square 4 miles on a side
- If you multiply an area in square feet by a height in feet, the result will have units of
  - feet.
  - feet<sup>2</sup>.
  - feet<sup>3</sup>.
- There are 1760 yards in a mile. Therefore, one cubic mile represents
  - 1760 square yards.
  - 1760<sup>3</sup> yards<sup>3</sup>.
  - 1760 yard-miles.
- The fact that 1 liter = 1.06 quarts can be written as the conversion factor
  - 1.06 quarts/liter.
  - 1.06 liters/quart.
  - 1 quart/1.06 liters.
- To end up with units of speed, you need to
  - multiply a distance by a time.
  - divide a distance by a time.
  - divide a time by a distance.
- If the current exchange rate is \$1.20 per euro, then
  - \$1 is worth more than 1 euro.
  - 1 euro is worth more than \$1.
  - 1 euro is equal to \$0.80.
- One square foot is equivalent to
  - 12 square inches.
  - 120 square inches.
  - 144 square inches.
- You are given two pieces of information: (1) the volume of a lake in cubic feet and (2) the average depth of the lake in feet. You are asked to find the surface area of the lake in square feet. You should
  - multiply the volume by the depth.
  - divide the volume by the depth.
  - divide the depth by the volume.
- You are given two pieces of information: (1) the price of gasoline in dollars per gallon and (2) the gas mileage of a car in miles per gallon. You are asked to find the cost of driving this car in dollars per mile. You should
  - divide the price of gas by the car's gas mileage.
  - multiply the price of gas by the car's gas mileage.
  - divide the car's gas mileage by the price of gas.

## Exercises 2A

### REVIEW QUESTIONS

- What are units? Describe how to read units that involve division, multiplication, squares, and cubes.
- Explain why a unit conversion really involves just multiplying by 1.
- Describe the three forms in which we can write any conversion factor. Given the conversion in one form, such as  $1 \text{ lb} = 16 \text{ oz}$ , how do you find the other two forms?
- Explain in words or with a picture why there are 9 square feet in 1 square yard and 27 cubic feet in 1 cubic yard. Then describe generally how to find conversion factors involving squares or cubes.
- Describe how to read and use the currency data in Table 2.1.
- Briefly describe how units can help you check your answers and solve problems. Give examples.

### DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

- I drove at a speed of 35 miles for the entire trip.
- Our house has a floor area of 1500 square feet.
- I have a box with a volume of 2 square feet.
- We own 2 square acres of farm land.
- I figured out how long the airplane will take to reach Beijing by dividing the airplane's speed by the distance to Beijing.
- I figured out how long the airplane will take to reach Beijing by dividing the distance to Beijing by the airplane's speed.

**BASIC SKILLS & CONCEPTS**

**13–18: Working with Fractions.** The following exercises require the skills covered in the Brief Review on pp. 84–85.

13. Evaluate the following expressions.

a.  $\frac{3}{4} \times \frac{1}{2}$     b.  $\frac{2}{3} \times \frac{3}{5}$     c.  $\frac{1}{2} + \frac{3}{2}$     d.  $\frac{2}{3} + \frac{1}{6}$

e.  $\frac{2}{3} \times \frac{1}{4}$     f.  $\frac{1}{4} + \frac{3}{8}$     g.  $\frac{5}{8} - \frac{1}{4}$     h.  $\frac{3}{2} \times \frac{2}{3}$

14. Evaluate the following expressions.

a.  $\frac{1}{3} + \frac{1}{5}$     b.  $\frac{10}{3} \times \frac{3}{7}$     c.  $\frac{3}{4} - \frac{1}{8}$     d.  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$

e.  $\frac{6}{5} + \frac{4}{15}$     f.  $\frac{3}{5} \times \frac{2}{7}$     g.  $\frac{1}{3} + \frac{13}{6}$     h.  $\frac{3}{5} \times \frac{10}{3} \times \frac{3}{2}$

15. Write each of the following as a common fraction.

a. 3.5    b. 0.3    c. 0.05    d. 4.1  
e. 2.15    f. 0.35    g. 0.98    h. 4.01

16. Write each of the following as a common fraction.

a. 2.75    b. 0.45    c. 0.005    d. 1.16  
e. 6.5    f. 4.123    g. 0.0003    h. 0.034

17. Convert the following fractions to decimal form; round to the nearest thousandth if necessary.

a.  $\frac{1}{4}$     b.  $\frac{3}{8}$     c.  $\frac{2}{3}$     d.  $\frac{3}{5}$   
e.  $\frac{13}{2}$     f.  $\frac{23}{6}$     g.  $\frac{103}{50}$     h.  $\frac{42}{26}$

18. Convert the following fractions to decimal form; round to the nearest thousandth if necessary.

a.  $\frac{1}{5}$     b.  $\frac{4}{9}$     c.  $\frac{4}{11}$     d.  $\frac{12}{7}$   
e.  $\frac{28}{9}$     f.  $\frac{56}{11}$     g.  $\frac{102}{49}$     h.  $\frac{15}{4}$

**19–26: Identifying Units.** Identify the units of the following quantities. State the units mathematically (for example, mi/hr) and in words (for example, miles per hour).

19. Your average speed on a long walk, found by dividing distance traveled in miles by time elapsed in hours
20. The unit price of oranges, found by dividing the price in dollars by the weight in pounds
21. The cost of a piece of carpet, found by dividing its price in dollars by its area in square yards
22. The flow rate of a river in which 5000 cubic feet of water flow past a particular location every second
23. The unit price of rice in Japan, found by dividing the price in yen by the weight in kilograms
24. The production rate of a bagel bakery, found by dividing the number of bagels produced by the time required in hours

25. The per capita daily oil consumption by the residents of a town, found by dividing the amount of oil used per day in gallons by the population of the town

26. The density of a rock, found by dividing its weight in grams by its volume in cubic centimeters

**27–36: Unit Conversions.** Carry out the following unit conversions.

27. Convert 24 feet to inches.
28. Convert 24 feet to yards.
29. Convert 25 minutes to seconds.
30. Convert 2.5 hours to seconds.
31. Convert 15 gallons to quarts.
32. Convert 4 acres to units of square feet.
33. Convert 140 weeks to days.
34. Convert 60 miles per hour to units of miles per minute.
35. Convert 3 years to hours (neglecting leap years).
36. Convert 1 ton (2000 pounds) to ounces.

**37. Area and Volume Calculations.** Show clearly your use of units for the following calculations.

a. A large box-shaped arena has a rectangular floor that measures 200 feet by 150 feet, and a flat ceiling that is 35 feet above the floor. Find the area of the floor and the volume of the arena.

b. A flat-bottom reflecting pool has length 30 yards, width 10 yards, and depth 0.3 yard. Find the surface area of the pool and the volume of water it holds.

c. A raised flower bed is 25 feet long, 8 feet wide, and 1.5 feet deep. Find the area of the bed and the volume of soil it holds.

**38. Area and Volume Calculations.** Clearly show the use of units in the following exercises.

a. A warehouse is 40 yards long and 25 yards wide, and it is piled with cartons to a height of 3 yards. What is the area of the warehouse floor? What is the total volume of the cartons? (Assume there is no space between cartons.)

b. The bed of a truck is 3.5 feet deep, 12 feet long, and 5 feet wide. What is the area of the bed's floor? What is the volume of the bed?

c. A can has a circular base with an area of 6 square inches and is 4 inches tall. What is its total volume?

**39–46: Conversions with Square and Cubic Units.**

39. Find a conversion factor between square feet and square inches. Write it in three forms.
40. Find a conversion factor between cubic feet and cubic inches. Write it in three forms.
41. Given that 1 meter = 100 centimeters, find a conversion factor between square meters and square centimeters. Write it in three forms.
42. Find the area in square feet of a rectangular yard that measures 20 yards by 12 yards.

43. A new sidewalk will be 4 feet wide, 200 feet long, and filled to a depth of 6 inches (0.5 foot) with concrete. How many cubic yards of concrete are needed?
44. How many cubic inches are there in 2.3 cubic yards?
45. An air conditioning system can circulate 320 cubic feet of air per minute. How many cubic yards of air can it circulate per minute?
46. A hot tub pump circulates 4 cubic feet of water per minute. How many cubic inches of water does it circulate each minute?
- 47–54: Currency Conversions.** Use the currency exchange rates in Table 2.1 for the following questions.
47. You arrive in London with \$400. How many pounds can you buy?
48. As you leave Paris, you convert 4500 euros to dollars. How many dollars do you receive?
49. You return from Mexico with 3000 pesos. How much are they worth in U.S. dollars?
50. Which is worth more: \$1 U.S. or \$1 Canadian?
51. Which is worth more: 1 yen or 1 peso?
52. Bottled water costs 0.75 euro per liter in Paris. What is the price in units of U.S. dollars per quart? (1 quart = 0.9464 liter)
53. Gasoline sells for 1.2 euros/liter in Bonn. What is the price in U.S. dollars per gallon? (1 gallon = 3.785 liters)
54. Cantaloupes sell for 1.80 euros per kilogram in Belgium. What is the price in units of U.S. dollars per pound? (1 kilogram = 2.205 pounds)
- 55–68: Working with Units.** Use unit conversions to answer the following questions.
55. An airliner travels 45 miles in 5 minutes. What is its speed in miles per hour?
56. What is the total cost of 1.2 cubic yards of soil if it sells for \$24 per cubic yard?
57. A hose fills a hot tub at a rate of 3.2 gallons per minute. How many hours will it take to fill a 300-gallon hot tub?
58. Competition speed skydivers have reached record speeds of 614 miles per hour. At this speed, how many feet would you fall every second?
59. How much would you pay for 2.5 ounces of gold at a price of \$920 per ounce?
60. Suppose you earn \$8.50 per hour and work 24 eight-hour days in a month. How much do you earn in that month?
61. In 2008, 565,650 Americans died of (all forms of) cancer. Assuming a population of 305 million, what was the mortality rate in units of deaths per 100,000 people?
62. In 2008, about 310,000 Americans died of sudden cardiac death (about half of all deaths from coronary heart disease). Assuming a population of 305 million, what was the mortality rate in units of deaths per 100,000 people?
63. There are approximately 3 million births in the United States each year. Find the birth rate in units of births per minute.
64. During a long road trip, you drive 420 miles on a 12-gallon tank of gas. What is your gas mileage (in miles per gallon)?
65. If your car gets 28 miles per gallon, how much does it cost to drive 250 miles when gasoline costs \$2.90 per gallon?
66. The median salary for the New York Yankees in 2008 was \$1,875,000. Assuming a 160-game season, express this salary in dollars per game.
67. If you sleep an average of 8 hours each night, how many hours do you sleep in a year?
68. A human heart beats about 70 times per minute. If an average human being lives to the age of 80, how many times does the average heart beat in a lifetime?
- 69–72: What Went Wrong?** Consider the following exam questions and student solutions. Determine whether the solution is correct. If it is not correct, write a note to the student explaining why it is wrong and give a correct solution.
69. *Exam Question:* A candy store sells chocolate for \$7.70 per pound. The piece you want to buy weighs 0.11 pound. How much will it cost, to the nearest cent? (Neglect sales tax.)  
*Student Solution:*  $0.11 \div 7.70 = 0.014$ . It will cost 1.4¢.
70. *Exam Question:* You ride your bike up a steep mountain road at 5 miles per hour. How far do you go in 3 hours?  
*Student Solution:*  $5 \div 3 = 1.7$ . I ride 1.7 miles.
71. *Exam Question:* You can buy a 50-pound bag of flour for \$11 or you can buy a 1-pound bag for \$0.39. Compare the per pound cost for the large and small bags.  
*Student Solution:* The large bag price is  $50 \div \$11 = \$4.55$  per pound, which is much more than the 39¢ per pound price of the small bag.
72. *Exam Question:* The average person needs 1500 Calories a day. A can of Coke contains 140 Calories. How many Cokes would you need to drink to fill your daily caloric needs? (Note: This diet may not meet other nutritional needs!)  
*Student Solution:*  $1500 \times 140 = 210,000$ . You would need to drink 210,000 Cokes to meet your daily caloric needs.
- 73–76: Gas Mileage.** Answer the following practical gas mileage questions.
73. You plan to take a 2000-mile trip in your car, which averages 32 miles per gallon. How many gallons of gasoline should you expect to use? Would a car that has only half the gas mileage (16 miles per gallon) require twice as much gasoline for the same trip? Explain.
74. Two friends take a 3000-mile cross-country trip together, but they drive their own cars. Car A has a 12-gallon gas tank and averages 40 miles per gallon, while car B has a 20-gallon gas tank and averages 30 miles per gallon. Assume both drivers pay an average of \$2.90 per gallon of gas.
- What is the cost of one full tank of gas for car A? For car B?
  - How many tanks of gas do cars A and B each use for the trip?
  - How much do the drivers of cars A and B each pay for gas for the trip?

75. Gas mileage actually varies slightly with the driving speed of a car (as well as with highway vs. city driving). Suppose your car averages 38 miles per gallon on the highway if your average speed is 55 miles per hour, and it averages 32 miles per gallon on the highway if your average speed is 70 miles per hour.
- What is the driving time for a 2000-mile trip if you drive at an average speed of 55 miles per hour? What is the driving time at 70 miles per hour?
  - Assume a gasoline price of \$2.90 per gallon. What is the gasoline cost for a 2000-mile trip if you drive at an average speed of 55 miles per hour? What is the gasoline cost at 70 miles per hour?
76. Suppose your car averages 32 miles per gallon on the highway if your average speed is 60 miles per hour, and it averages 25 miles per gallon on the highway if your average speed is 75 miles per hour.
- What is the driving time for a 1500-mile trip if you drive at an average speed of 60 miles per hour? What is the driving time at 75 miles per hour?
  - Assume a gasoline price of \$2.90 per gallon. What is the gasoline cost for a 1500-mile trip if you drive at an average speed of 60 miles per hour? What is the gasoline cost at 75 miles per hour?
77. **Greenland Ice Sheet.** The Greenland ice sheet contains about 3 million cubic kilometers of ice. If completely melted, this ice would release about 2.5 million cubic kilometers of water, which would spread out over Earth's 340 million square kilometers of ocean surface. How much would sea level rise?
78. **Practical Carpet Problem.** Suppose you want to install carpet in a room that measures 18 feet by 22 feet. The carpet you want costs \$28.50 per square yard and comes only in rolls that are 12 feet wide (and at least 100 feet long). If you allow only one seam (where two pieces of carpet meet), what is the most efficient way to lay the carpet and how much will the carpet cost?

### FURTHER APPLICATIONS

79. **Professional Basketball Salaries.** In the 2009 season, Kevin Garnett of the Boston Celtics earned \$24,750,000 to play 80 games, each lasting 48 minutes. (Assume no overtime games.)
- How much did Garnett earn per game?
  - Assuming that Garnett played every minute of every game, how much did he earn per minute?
  - Assuming that Garnett played  $\frac{2}{3}$  of every game, how much did he earn per minute?
  - Suppose that, averaged over the course of a year, Garnett practiced or trained 40 hours for every game. Including this training time, what was his hourly salary?
80. **Full of Hot Air.** The average person breathes 6 times per minute (at rest), inhaling and exhaling half a liter of air each time. How much "hot air" (air warmed by the body), in liters, does the average person exhale each day?
81. **Kindle 2.** Amazon's Kindle 2 wireless reading device has a capacity of 1.4 gigabytes (1.4 billion bytes). Assume that one byte corresponds to one character and that an average page consists of 2000 characters.
- How many pages of text can one Kindle hold?
  - How many 500-page books can one Kindle hold?
82. **Volcanic Eruption.** The greatest volcanic eruption in recorded history took place in 1815 on the Indonesian island of Sumbawa, when the volcano Tambora expelled an estimated 100 cubic kilometers of molten rock. Suppose all of the ejected material fell on a region with an area of 600 square kilometers. Find the average depth of the resulting layer of ash and rock.
83. **Shower vs. Bath.** Assume that when you take a bath, you fill a tub to the halfway point and the tub measures 6 feet by 3 feet by 2.5 feet. When you take a shower, you use a shower head with a flow rate of 1.75 gallons per minute, and you typically spend 10 minutes in the shower. There are 7.5 gallons in one cubic foot.
- Do you use more water taking a shower or taking a bath?
  - How long would you need to shower in order to use as much water as you use taking a bath?
  - Assuming your shower is in a bath tub, propose a nonmathematical way to compare, in one experiment, the amounts of water you use taking a shower and a bath.
84. **Hurricane Katrina.** Experts estimate that when the levees around New Orleans broke in the aftermath of Hurricane Katrina, water flowed into the city at a peak rate of 9 billion gallons per day. There are 7.5 gallons in one cubic foot.
- Find the flow rate in units of cubic feet per second (cfs). Compare this flow rate to the average flow rate of the Colorado River in the Grand Canyon: 30,000 cfs.
  - Assume that the flooded part of the city had an area of 6 square miles. Estimate how much (in feet) the water level rose in one day at the given flow rate.
85. **Glen Canyon Flood.** The Department of the Interior periodically releases a "spike flood" from the Glen Canyon Dam into the Colorado River. The purpose is to restore the river and the habitats along its banks, particularly in the Grand Canyon. The reservoir behind the dam (Lake Powell) holds approximately 1.2 trillion (1,200,000,000,000) cubic feet of water. During a recent week-long spike flood, water was released at a rate of 25,800 cubic feet per second. How much water was released during the 1-week flood? What percentage of the total water in the reservoir was released during the flood?

