

9. You currently earn \$1000 per month, but you are expecting your earnings to rise 10% per year. This means that in five years you expect to be earning
- somewhat less than \$1500 per month.
 - exactly \$1500 per month.
 - somewhat more than \$1500 per month.
10. During high school, Elise won 30% of the swim races she entered. During college, Elise won 20% of the swim races she entered. We can conclude that, in high school and college combined, Elise won
- 25% of the races she entered.
 - more than 20% but less than 30% of the races she entered.
 - more than 26% but less than 28% of the races she entered.

Exercises 3A

REVIEW QUESTIONS

- Describe the three basic uses of percentages. Give a sample statement that uses percentages in each of the three ways.
- Distinguish between absolute and relative change. Give an example that illustrates how we calculate a relative change.
- Distinguish between absolute and relative difference. Give an example that illustrates how we calculate a relative difference.
- Explain the difference between the key words *of* and *more than* when dealing with percentages. How are their meanings related?
- Explain the difference between the terms *percent* (%) and *percentage points*. Give an example of how they can differ for the same situation.
- Give an example to explain why, in general, it is not legitimate to average percentages.

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

- In many European countries, the percentage change in population has been negative in recent decades.
- The price of tuition has tripled since my parents went to school—that's a 200% increase in price!
- My older child weighs 25% more than my younger child.
- I've decreased my caloric intake by 125%, which has helped me lose weight.
- If you earn 20% more than I do, then I must earn 20% less than you do.
- If they raise taxes by 10% every year, in a decade we'll be paying everything we earn to taxes.
- We found that these rare cancers were 700% more common in children living near the toxic landfill than in the general population.
- I have a 60% average on our assignments going into the final exam, but I still hope to raise my course average to 70% by getting an 80% on the final. (The final is worth 25% of the final grade.)
- The rate of return on our fund increased by 50%, to 15%.
- My bank increased the interest rate on my savings account 100%, from 2% to 4%.

BASIC SKILLS & CONCEPTS

17–32: **Fractions, Decimals, Percentages.** Express the following numbers in three forms: as a reduced fraction, as a decimal, and as a percentage. (Refer to the Brief Review on p. 129.)

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|-------------------|-------------------|-------------------|---------------|
| 17. $\frac{2}{5}$ | 18. 30% | 19. 0.20 | 20. 0.85 |
| 21. 150% | 22. $\frac{2}{3}$ | 23. $\frac{4}{9}$ | 24. 1.25 |
| 25. $\frac{5}{8}$ | 26. 44% | 27. 69% | 28. 4.25 |
| 29. $\frac{7}{5}$ | 30. 121% | 31. $\frac{4}{3}$ | 32. 0.6 . . . |

33–44: **Review of Ratios.** Compare the following pairs of numbers *A* and *B* in three ways:

- Find the ratio of *A* to *B*.
 - Find the ratio of *B* to *A*.
- c. Complete the sentence: *A* is _____ percent of *B*. (Refer to the Brief Review on p. 134.)
- $A = 8$ and $B = 4$
 - $A = 150$ and $B = 400$
 - $A = 52,252$ is the number of deaths due to AIDS in the United States in 1995 and $B = 16,268$ is the number of deaths due to AIDS in the United States in 2005 (most recent data available).
 - $A = 40,229,000$ is the number of Americans over age 65 in 2010 and $B = 88,458,000$ is the projected number of Americans over age 65 in 2050.
 - $A = 3.9$ million is the population of Los Angeles and $B = 2.9$ million is the population of Chicago.
 - $A = 466$ is the average mathematics SAT score in Maine and $B = 523$ is the average mathematics SAT score in Vermont.
 - $A = 2,290,000$ is the 2009 daily circulation of *USA Today* (highest in the nation) and $B = 2,010,000$ is the 2009 daily circulation of the *Wall Street Journal* (second highest in the nation).
 - $A = \$87,000$ is the median price of a single-family home in Davenport, Iowa, in 2008 and $B = \$636,000$ is the median price of a single-family home in Honolulu, Hawaii, in 2008.
 - $A = 100$ square meters is the area of a square with sides of length 10 meters and $B = 25\pi$ square meters is the area of a circle with a diameter of 10 meters.
 - $A = 87.8\%$ is the high school graduation rate in Nebraska (first in the nation) and $B = 55.8\%$ is the high school graduation rate in Nevada (last in the nation).

45–50: Percentages as Fractions. In the following statements, express the first number as a percentage of the second number.

45. 28 pounds of recyclable trash in a barrel of 52 pounds of trash
46. 12.0 million metric tons of beef produced annually in the United States out of 65.1 million metric tons of beef produced annually worldwide
47. The median salary for U.S. men in 2007 was \$33,196 and the median salary for U.S. women in 2007 was \$20,922 (most recent data).
48. 40.4 million Americans under age 15 out of 305 million Americans
49. 70.3 Americans per square mile in 1990 and 86.4 Americans per square mile in 2010
50. An estimated 10,000 U.S. nuclear weapons out of an estimated 27,000 nuclear weapons worldwide (as of 2010)
51. **Salary Comparisons.** Clint's salary increased from \$20,000 to \$28,000 over a three-year period. Helen's salary increased from \$25,000 to \$35,000 over the same period. Whose salary increased more in absolute terms? In relative terms? Explain.
52. **Population Comparison.** Between 2000 and 2010, the population of Las Vegas increased from 478,000 to 565,000. Over the same decade, the population of Dallas increased from 1,189,000 to 1,245,000. Which city had the greater absolute growth in population? Which city had the greater percentage growth in population?

53–56: Percentage Change. Find the absolute change and the percentage change in the following cases.

53. The population of the United States increased from 249 million in 1990 to 308 million in 2010.
54. The congressional delegation of California increased from 30 in 1950 to 53 in 2010.
55. The number of daily newspapers in the United States was 2226 in 1900 and 1420 in 2010.
56. The number of music CDs shipped in the United States decreased from 942 million in 2000 to 511 million in 2008.

57–62: Percentage Difference. Complete the following sentences.

57. The gestation period of humans (266 days) is _____ percent longer than the gestation period of grizzly bears (220 days).
58. The 2009 life expectancy in Canada (81.2 years) is _____ percent greater than the 2009 life expectancy in Russia (65.9 years).
59. The main span of the Tacoma Narrows bridge (2800 feet) is _____ percent shorter than the main span of the Golden Gate bridge (4200 feet).
60. The median age at first marriage of U.S. women (25.9 years) is _____ percent less than the median age at first marriage of U.S. men (27.5 years).
61. The number of deaths due to poisoning in the United States in a year (25,500) is _____ percent greater than the number of deaths due to falls (21,400).

62. The amount of rice produced by China in 2007 (190 million metric tons) was _____ percent greater than the amount of rice produced by India in 2007 (127 million metric tons).

63–66: Of vs. More Than. Fill in the blanks in the following statements.

63. Will is 22% taller than Wanda, so Will's height is _____ % of Wanda's height.
64. The area of Norway is 24% more than the area of Colorado, so Norway's area is _____ % of Colorado's area.
65. The population of Virginia is 18% less than the population of Georgia, so Virginia's population is _____ % of Georgia's population.
66. The net worth of Zelda is 6.4% less than the net worth of Alicia, so Zelda's net worth is _____ % of Alicia's net worth.

67–70: Prices and Sales. Fill in the blanks in the following statements.

67. The wholesale price of a TV is 40% less than the retail price. Therefore, the wholesale price is _____ times the retail price.
68. A store is having a 50% off sale. Therefore, the original price of an item is _____ times as much as the sale price.
69. The retail cost of a TV is 30% more than its wholesale cost. Therefore, the retail cost is _____ times the wholesale cost.
70. A store is having a 40% off sale. The sale price for an item with a regular cost of \$80 is _____.

71–74: Percentages of Percentages. Describe each of the following changes in two ways: as an absolute change in terms of percentage points and as a relative change in terms of a percentage.

71. The annual interest rate for Jack's savings account increased from 2.3% to 2.8%.
72. The percentage of Republicans in the House of Representatives decreased from 53.3% in 2007 to 46.4% in 2009.
73. The percentage of Americans accessing the Internet increased from 66.9% in 2000 to 78.5% in 2010.
74. The percentage of fifth-graders who scored at or above the national average on the reading test rose to 44 percent from 25 percent.
75. **Care in Wording.** Assume that 30% of city employees in Carson City ride the bus to work. Consider the following two statements:

- The percentage of city employees who ride the bus to work is 10% higher in Freetown than in Carson City.
- The percentage of city employees who ride the bus to work is 10 percentage points higher in Freetown than in Carson City.

For each case, state the percentage of city employees in Freetown who ride the bus to work. Briefly explain why the two statements have different meanings.

76. **Ambiguous News.** The average annual precipitation on Mt. Washington, New Hampshire, is 90 inches. During one

particularly wet year, different news reports carried the following statements.

- The precipitation this year is 200% of normal.
- The precipitation this year is 200% above normal.

What does each of these statements imply about the precipitation during this year? Do the two statements have the same meaning? Explain.

77–84: Solving Percentage Problems. Solve the following percentage problems.

77. You purchase a bicycle with a retail (pre-tax) price of \$760. The local sales tax rate is 7.6%. What is the final cost?
78. The final cost of your new shoes is \$107.69. The local sales tax rate is 6.2%. What was the retail (pre-tax) price?
79. The 2410 women undergraduates at the college comprise 54% of all undergraduates. How many undergraduates attend the college?
80. The number of housing foreclosures in 2008 was 3.98 million, up 81% over 2007. How many foreclosures were there in 2007?
81. Your total cost for dinner in a restaurant after the tip was \$76.40. Assuming you left a 20% tip, what was the bill for food and taxes?
82. Your dinner bill is \$42.50. You leave \$50. What percent tip did you leave? (Ignore taxes in this question.)
83. Simon's monthly take-home pay (after taxes) is \$2200. If he pays 21% of his gross pay (before taxes) in tax, what is his gross pay?
84. Eileen earned \$45 in annual interest from a savings account with a 3.5% annual interest rate. Assuming she made no deposits or withdrawals during the year, what is the balance in the account?

85–88: Shifting Reference Value. State whether the following statements are true or false, and explain why. If a statement is false, state the true change.

85. If the national economy shrank at an annual rate of 4% per year for three consecutive years, then the economy shrank by 12% over the three-year period.
86. You receive a pay raise of 5%, then receive a pay cut of 5%. After the two changes in pay, your salary is unchanged.
87. If the profits in your consulting business increase by 4% one year and decrease by 2% the following year, your profits are up by 2% over two years.
88. A high school reports that its students' SAT scores were down by 20% for one year. The next year, however, SAT scores rose by 30%. The high school principal announces, "Overall, test scores have improved by 10% over the past two years."

89–94: Is It Possible? Determine whether the following claims could be true. Explain your answer.

89. By turning off her lights and closing her windows at night, Maria saved 120% on her monthly energy bill.
90. Scott is 140% taller than his son.
91. Restaurant prices have increased 100% in the last 20 years.

92. Through hard training, Renee improved her 10-kilometer running time by 100%.
93. Your computer is 500% faster than mine.
94. Your computer is 250% slower than mine.
95. **Average Percentages.** You are a teacher. Your first-period class, with 25 students, had a mean score of 86% on the midterm exam. Your second-period class, with 30 students, had a mean score of 84% on the same exam. Does it follow that the mean score for both classes combined is 85%? Explain.
96. **Average Percentages.** A player has a batting average over many games of .400. In his next game, he goes 2 for 4, which is a batting average of .500 for the game. Does it follow that his new batting average is $(.400 + .500)/2 = .450$? Explain.

FURTHER APPLICATIONS

97–100: Analyzing Percentage Statements. Assuming the given information is accurate, determine whether the following statements are true. Provide an explanation.

97. The class is 60% women and 10% of the women have blond hair, so blond women comprise $60\% \times 10\% = 6\%$ of the class.
98. The class is 40% men and 20% of the men are bald, so bald men comprise $40\% \times 20\% = 8\%$ of the class.
99. 70% of the hotels have a restaurant and 20% have a swimming pool, so 90% of the hotels in town have a restaurant or a pool.
100. 40% of the class drives to school and 15% takes a bus to school, so 45% of the class neither drives nor takes a bus.

101–106: Percentages in the News. Answer the question that follows each quote from a news source.

101. "Some 80% of the estimated \$62 trillion in credit default swaps outstanding in 2008 were speculative." What is the value of the speculative credit default swaps?
102. "... the median annual income for adults with bachelor's degrees was \$37,729, compared with \$20,718 for those with only a high school diploma." The annual income for adults with bachelor's degrees is what percent greater than that for adults with a high school diploma only?
103. "The unemployment rate has risen more than a percentage point, to 8.5% in February from 7.1% last November." What is the relative change in the unemployment rate expressed as a percentage?
104. "The American Booksellers Association, which represents independent booksellers, includes 1500 businesses at 2500 locations. Twenty years ago it represented 4700 businesses at 5500 locations." What is the percent change in the number of businesses and locations represented?
105. "At the Gates Family Foundation, investment losses last year were 43 percent, or \$222 million of its endowment." How much was the endowment worth before the losses?