Exercises 3B

REVIEW QUESTIONS

1. Briefly describe scientific notation. How is it useful for writing large and small numbers? How is it useful for making approximations?
2. Explain how we can use estimation to put numbers in perspective. Give an example.
3. What is an order of magnitude estimate? Explain why such an estimate can be useful even though it may be as much as 10 times too large or too small.
4. Explain how we can use comparisons to put numbers in perspective. Give an example.
5. Describe three common ways of expressing the scale of a map or model. How would you show a scale of 1 cm = 100 km graphically? How would you describe it as a ratio?
6. Explain how we can use scaling to put numbers in perspective. Give an example.
7. Suppose that the Sun were the size of a grapefruit. How big and how far away would the Earth be on this scale? How far would the nearest stars (besides the Sun) be?
8. Describe several ways of putting each of the following in perspective: the size of a large university; $1 billion; the size of an atom; the Sun's remaining lifetime.

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

9. I read a book that had 10^5 words in it.
10. I've seen about 10^{50} commercials on TV.
11. I live in an apartment building that is 200 feet tall.
12. In total, Americans spend about a billion dollars per year on housing costs (rent and home mortgage payments).
13. After a recent NFL football game, the star player signed autographs for every person in attendance.
14. The CEO of the company earned more money last year than the company's 500 lowest paid employees combined.

BASIC SKILLS & CONCEPTS

15–20: Review of Scientific Notation. In the following exercises, use the skills covered in the Brief Review on p. 145.

15. Convert each of the following numbers from scientific to ordinary notation and write its name.

Example: $2 \times 10^3 = 2000 =$ two thousand

a. 3×10^3 b. 6×10^6 c. 3.4×10^5
d. 2×10^{-2} e. 2.1×10^{-4} f. 4×10^{-5}

16. Convert each of the following numbers from scientific to ordinary notation and write its name.

a. 8×10^2 b. 5×10^3 c. 9.6×10^4
d. 2×10^{-3} e. 3.3×10^{-5} f. 7.66×10^{-2}

17. Write each of the following numbers in scientific notation.

a. 233 b. 126,547 c. 0.11
d. 9736.23 e. 124.58 f. 0.8642

18. Write each of the following numbers in scientific notation.

a. 4327 b. 984.35 c. 0.0045
d. 624.87 e. 0.1357 f. 98.180004

19. Do the following operations without a calculator and show your work clearly. Be sure to express answers in scientific notation. You may round your answers to one decimal place (as in 3.2×10^5).

a. $(3 \times 10^3) \times (2 \times 10^2)$ b. $(4 \times 10^2) \times (3 \times 10^8)$
c. $(3 \times 10^3) + (2 \times 10^2)$ d. $(8 \times 10^{12}) \div (4 \times 10^4)$

20. Do the following operations without a calculator and show your work clearly. Be sure to express answers in scientific notation. You may round your answers to one decimal place (as in 3.2×10^5).

a. $(4 \times 10^7) \times (2 \times 10^8)$ b. $(3.2 \times 10^5) \times (2 \times 10^4)$
c. $(4 \times 10^3) + (5 \times 10^2)$ d. $(9 \times 10^{13}) \div (3 \times 10^{10})$

21–22: They Don't Look That Different! Compare the numbers in each pair and give the factor by which the numbers differ. *Example:* 10^6 is 10^2, or 100, times as large as 10^4.

21. a. 10^{35}, 10^{26} b. 10^{17}, 10^{27}
c. 1 billion, 1 million d. 7 trillion, 7 thousand
e. 2×10^{-6}, 2×10^{-9} f. 6.1×10^{27}, 6.1×10^{29}

22. a. 250 million, 5 billion b. 9.3×10^2, 3.1×10^{-2}
c. 10^{-8}, 2×10^{-13} d. 3.5×10^{-2}, 7×10^{-8}
e. 1 thousand, 1 thousandth f. 10^{12}, 10^{-9}

23–28: Using Scientific Notation. Rewrite the following statements using a number in scientific notation.

23. My new music player has a capacity of 340 gigabytes. (Recall that *giga* means billion.)
24. The estimated (2009) world nuclear energy production is 2608 billion kilowatt-hours.
25. As of 2009, U.S. consumer credit (total amount borrowed) was $2564 billion.
26. The number of different eight-character passwords that can be made with 26 letters and 10 numerals is approximately 2.8 trillion.
27. The diameter of a typical bacterium is about 0.000001 meter.
28. A beam of light can travel the length of a football field in about 30 nanoseconds. Express your answer in seconds. (*Hint*: Recall that *nano* means one billionth.)

29–30: Approximation with Scientific Notation. Estimate the following quantities without using a calculator. Then find the exact result, using a calculator if necessary. Discuss whether your approximation technique worked.

29. a. $300{,}000 \times 100$
   b. 5.1 million × 1.9 thousand
   c. $4 \times 10^9 \div 2.1 \times 10^6$
   d. 33 million × 3.1 thousand
   e. $4{,}288{,}364 \div 2132$
   f. $(6.129845 \times 10^6) \div (2.198 \times 10^4)$

30. a. 5.6 billion ÷ 200
   b. 4 trillion ÷ 260 million
   c. $9000 \times 54{,}986$
   d. 3 billion ÷ 25,000
   e. 5987×341
   f. $43 \div 765$

31–34: Perspective Through Estimation. Use estimation to make the following comparisons. Discuss your conclusion.

31. Which is greater: the amount you spend in a month on coffee or the amount you spend in a month on gasoline?
32. Could a person walk across the United States (New York to California) in a year? If not, about how long would it take?
33. Which is greater: the time you spend in a year studying or the time you spend in a year sleeping?
34. Which is greater: the amount you spend in a year on transportation or the amount you spend in a year on food?

35–40: Order of Magnitude Estimates. Make order of magnitude estimates of the following quantities. Explain the assumptions you use in your estimates.

35. The number of times your heart beats in a day
36. The number of steps you take in an average day
37. The amount of water you drink in a year
38. The total number of words in this textbook
39. The number of miles you travel in a car in a year (as either driver or passenger)
40. The number of hours per month that you listen to music

41–48: Energy Comparisons. Use Table 3.1 to answer the following questions.

41. How many average candy bars would you have to eat to supply the energy needed for 6 hours of running?
42. How many liters of oil are required to supply the electrical energy needs of an average home for a month?
43. Compare the energy released by burning 1 kilogram of coal to that released by fission of 1 kilogram of uranium-235.
44. Compare the energy released by burning 1 liter of oil to that released by fusion of the hydrogen in 1 liter of water.
45. If you could generate energy by fusing the hydrogen in ordinary tap water, how much water would you need to generate the electrical energy used daily by a typical home?
46. If you could generate energy by fusing the hydrogen in ordinary tap water, how much water would you need to supply all the energy currently consumed worldwide?
47. How many kilograms of uranium would be required to supply the energy needs of the United States for 1 year using fission?
48. Suppose that we could somehow capture all the energy released by the Sun for just 1 second. Would this energy be enough to supply U.S. energy needs for a year? Explain.

49–52: Scale Ratios. Find the scale ratios for the following maps.

49. 2 centimeters on the map represents 100 kilometers.
50. 1 inch on the map represents 10 miles.
51. 1 cm (map) = 500 km (actual)
52. 0.5 in. (map) = 1 km (actual)
53. **Scale Model Solar System.** The following table gives size and distance data for the planets. Calculate the scaled size and distance for each planet using a 1 to 10 billion scale model solar system. Give your results in table form. Then write one or two paragraphs that describe your findings in words and give perspective to the size of our solar system.

Planet	Diameter	Average Distance from Sun
Mercury	4880 km	57.9 million km
Venus	12,100 km	108.2 million km
Earth	12,760 km	149.6 million km
Mars	6790 km	227.9 million km
Jupiter	143,000 km	778.3 million km
Saturn	120,000 km	1427 million km
Uranus	52,000 km	2870 million km
Neptune	48,400 km	4497 million km

54. **Interstellar Travel.** The fastest spaceships launched to date are traveling away from Earth at speeds of about 50,000 kilometers per hour. How long would such a spaceship take to reach Alpha Centauri? (*Hint:* See Example 8.) Based on your answer, write one or two paragraphs discussing whether interstellar travel is a realistic possibility today.
55. **Universal Time Line.** According to modern science, Earth is about 4.5 billion years old and written human history extends back about 10,000 years. Suppose you represent the entire history of Earth with a 100-yard-long timeline, with the birth of Earth on one end and today at the other end.
   a. What distance represents 1 billion years?
   b. How far from the end of the timeline does written human history begin?
56. **Universal Clock.** According to modern science, Earth is about 4.5 billion years old and written human history extends back about 10,000 years. Suppose you represent the entire history of Earth by 12 hours on a clock, with the birth of Earth at the stroke of midnight and today at the stroke of noon.
   a. How much time on the clock represents 1 billion years?
   b. At what time on the clock does written human history begin?

FURTHER APPLICATIONS

57–64: Making Numbers Understandable. Restate the following facts as indicated.

57. There are approximately 2.4 million deaths per year in the United States. Express this quantity in deaths per minute.

58. There are approximately 2.2 million marriages per year in the United States. Express this quantity in marriages per hour.

59. Approximately 40,000 Americans are killed each year in automobile accidents. Express this quantity in deaths per day.

60. In 2007, Walmart had profits of $12.7 billion. Express this profit in terms of dollars per minute.

61. There were 305 million crimes (either violent or property crimes) committed in the United States in 2008. Express this quantity in terms of crimes per second and crimes per American.

62. The 2008 prison population in the United States was approximately 1.6 million. How many arenas with a capacity of 50,000 would this population fill?

63. Americans consume an estimated 7.6 billion pounds of candy (excluding chewing gum) per year. Express this quantity in terms of pounds per person per month. (Use a population of 305 million.)

64. In 2007, Procter and Gamble spent $5.2 billion on advertising. Express this quantity in terms of the height in kilometers of a stack of $1 bills. Assume 10 bills per millimeter.

65. **Cells in the Human Body.** Estimates of the number of cells in the human body vary over an order of magnitude. Indeed, the precise number varies from one individual to another and depends on whether you count bacterial cells. Here is one way to make an estimate.

a. Assume that an average cell has a diameter of 6 micrometers (6×10^{-6} meter), which means it has a volume of 100 cubic micrometers. How many cells are there in a cubic centimeter?

b. Estimate the number of cells in a liter, using the fact that a cubic centimeter equals a milliliter.

c. Estimate the number of cells in a 70-kilogram (154-pound) person, assuming that the human body is 100% water (actually it is about 60–70% water) and that 1 liter of water weighs 1 kilogram.

66. **CO_2 Emissions.** For every gallon of gasoline burned by an automobile, approximately 10.2 kilograms of carbon dioxide are emitted into the atmosphere. Estimate the total amount of carbon dioxide added to the atmosphere by all automobile travel in the United States over the past year.

67. **The Amazing Amazon.** An issue of *National Geographic* contained the following statement:

> *Dropping less than two inches per mile after emerging from the Andes, the Amazon drains a sixth of the world's runoff into the ocean. One day's discharge at its mouth—4.5 trillion gallons—could supply all U.S. households for five months.*

Based on this statement, determine how much water an average U.S. household uses each month. Does this answer seem reasonable? Explain any estimates you make.

68. **Wood for Energy?** A total of about 180,000 terawatts of solar power reaches Earth's surface, of which about 0.06% is used by plants for photosynthesis. Of the energy that goes to photosynthesis, about 1% ends up stored in plant matter (including wood). (Recall that 1 watt = 1 joule/s; 1 terawatt = 10^{12} watts.)

a. Calculate the total amount of energy that becomes stored in plant matter each second.

b. Suppose that power stations generated electricity by burning plant matter. If all the energy stored in plants could be converted to electricity, what average power, in terawatts, would be possible? Would it be enough to meet world electricity demand, which is of order 10 terawatts?

c. Based on your answer to b, can you draw any conclusions about why humans depend on fossil fuels, such as oil and coal, which are the remains of plants that died long ago? Explain.

69. **Stellar Corpses: White Dwarfs and Neutron Stars.** A few billion years from now, after exhausting its nuclear engines, the Sun will become a type of remnant star called a *white dwarf.* It will still have nearly the same mass (about 2×10^{30} kg) as the Sun today, but its radius will be only about that of Earth (about 6400 km).

a. Calculate the average density of the white dwarf in units of kilograms per cubic centimeter.

b. What is the mass of a teaspoon of material from the white dwarf? (*Hint*: A teaspoon is about 4 cubic centimeters.) Compare this mass to the mass of something familiar (for example, a person, a car, a tank).

c. A neutron star is a type of stellar remnant compressed to even greater densities than a white dwarf. Suppose that a neutron star has a mass 1.4 times the mass of the Sun but a radius of only 10 kilometers. What is its density? Compare the mass of 1 cubic centimeter of neutron star material to the total mass of Mt. Everest (about 5×10^{10} kg).

70–73: Sampling Problems. Sampling techniques can be used to estimate physical quantities. To estimate a large quantity, you might measure a representative small sample and find the total quantity by "scaling up." To estimate a small quantity, you might measure several of the small quantities together and "scale down".

Example: How thick is a sheet of a paper?

Solution: One way to estimate the thickness of a sheet of paper is to measure the thickness of a ream (500 sheets) of paper. A particular ream was 7.5 centimeters thick. Thus, a sheet of paper from this ream was 7.5 cm ÷ 500 = 0.015 cm thick, or 0.15 millimeter.

70. How much does a sheet of paper weigh?

71. How thick is a penny? a nickel? a dime? a quarter? Would you rather have your height stacked in pennies, nickels, dimes, or quarters? Explain.

72. How much does a grain of sand weigh? How many grains of sand are in a typical playground sand box?

73. How many stars are visible in the sky on the clearest, darkest nights? How could astronomers estimate the total number of stars in the universe?

WEB PROJECTS

74. **Energy Comparisons.** Using data available from the Energy Information Administration Web site, choose a few measures of U.S. or world energy consumption or production. Make comparisons that put these numbers in perspective.

75. **Nuclear Fusion.** Learn about the current state of research into building commercially viable fusion power plants. What obstacles must still be overcome? Do you think fusion power will be a reality in your lifetime? Explain.

76. **Scale Model Solar System.** Visit the Web site for the Voyage scale model solar system on the National Mall in Washington, D.C. Write a brief report on what you learn.

77. **Richest People.** Find the net worth of the world's three richest people. Put these monetary values in perspective through any techniques you wish.

IN YOUR WORLD

78. **Large Numbers.** Search today's newspaper for as many numbers larger than 100,000 as you can find. Briefly explain the context within which each large number is used.

79. **Perspective in the News.** Find an example in the recent news in which a reporter uses a technique to put a number in perspective. Describe the example. Do you think the technique is effective? Can you think of a better way to put the number in perspective? Explain.

80. **Putting Numbers in Perspective.** Find at least two examples of very large or small numbers in recent news reports. Use a technique of your choosing to put each number in perspective in a way that you believe most people would find meaningful.

TECHNOLOGY EXERCISE

81. **Scientific Notation with Technology.** Use a calculator or Excel to do the following calculations.

a. Find the distance that light travels in a year at a speed of 186,000 miles/second (a distance called a *light-year*).

b. Evaluate $\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$, the number of different 5-card hands that can be dealt from a 52-card deck of cards.

c. Annual worldwide emissions of carbon dioxide are estimated to be 30,000 million metric tons (2009). Express this quantity in per capita terms (metric tons per person in the world). Assume a world population of 6.8 billion.

d. Earth's mass is 6.0×10^{24} kilograms. Its volume is 1.1×10^{12} cubic kilometers. Find the density of Earth (mass/volume) in units of grams per cubic centimeter. (For comparison, the density of water is 1 gram per cubic centimeter.)

e. The universe is estimated to be about 14 billion years old. What is its age in seconds?

UNIT 3C Dealing with Uncertainty

> None of us really understands what's going on with all these numbers.
>
> —David Stockman, Budget Director for President Reagan, 1981

The rosy outlook for the United States federal budget was big news in early 2001. Government economists projected a cumulative surplus of \$5.6 *trillion* for the coming decade. Politicians argued about whether this windfall should be returned to taxpayers, spent on new programs, or used to pay down the federal debt.

Two years later, the projected surplus not only had vanished, but had been replaced by huge deficits. By 2009, rather than having gained trillions in surplus money, the government had instead gone another \$6 trillion into debt. How did the nearly \$6 trillion surplus—some \$20,000 for every man, woman, and child in the United States—disappear?

It didn't really disappear, of course. Rather, it never existed in the first place—it was only an estimate. Like all estimates, it was only as good as the assumptions that went into it, and these assumptions included highly uncertain predictions about the future of the economy, future tax rates, and future spending. The government economists who made the projection were well aware of the uncertainties, but the news media and politicians tended to report the surplus projection as an indisputable fact.

This story of vanishing trillions holds an important lesson. Many of the numbers we encounter in daily life are far less certain than we are told, and we can therefore be severely misled unless we learn to examine and interpret uncertainties for ourselves. In this unit, we will discuss ways of dealing with uncertainty properly. Understanding the nature of uncertainty will make you better equipped to assess the reliability of numbers as they are typically reported in the news.