DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

5. Money in a bank account earning compound interest at an annual percentage rate of 3% is an example of exponential growth.
6. Suppose you had a magic bank account in which your balance doubled each day. If you started with just $1, you'd be a millionaire in less than a month.
7. A small town that grows exponentially can become a large city in just a few decades.
8. Human population has been growing exponentially for a few centuries, and we can expect this trend to continue forever in the future.

BASIC SKILLS & CONCEPTS

9–16: Linear or Exponential? State whether the growth (or decay) is linear or exponential, and answer the associated question.

9. The population of MeadowView is increasing at a rate of 325 people per year. If the population is 2500 today, what will it be in four years?
10. The population of Winesburg is increasing at a rate of 3% per year. If the population is 75,000 today, what will it be in three years?
11. During the 1999 episode of hyperinflation in Brazil, the price of food increased at a rate of 30% per month. If your food bill was R$100 one month during this period, what was it three months later? (R$ is the symbol for the *real*, Brazil's unit of currency.)
12. The price of a gallon of gasoline is increasing by 3¢ per week. If the price is $3.10 per gallon today, what will it be in ten weeks?
13. The price of computer memory is decreasing at a rate of 14% per year. If a memory chip costs $50 today, what will it cost in three years?
14. The value of your car is decreasing by 10% per year. If the car is worth $12,000 today, what will it be worth in two years?
15. The value of your house is increasing by $2000 per year. If it is worth $100,000 today, what will it be worth in five years?
16. The value of your house is decreasing by 7% per year. If it is worth $250,000 today, what will it be worth in three years?

17–20: Chessboard Parable. Use the chessboard parable presented in the text. Assume that each grain of wheat weighs 1/7000 pound.

17. How many grains of wheat should be placed on square 16 of the chessboard? Find the total number of grains and their total weight (in pounds) at this point.
18. How many grains of wheat should be placed on square 32 of the chessboard? Find the total number of grains and their total weight (in pounds) at this point.
19. What is the total weight of all the wheat when the chessboard is full?
20. According to the U.S. Department of Agriculture, the current world harvest of all wheat, rice, and corn is less than 2 billion tons per year. How does this total compare to the weight of the wheat on the chessboard? (*Hint*: 1 ton = 2000 pounds.)

21–24: Magic Penny Parable. Use the magic penny parable presented in the text.

21. How much money would you have after 22 days?
22. Suppose that you stacked the pennies after 22 days. How high would the stack rise, in kilometers? (*Hint*: Find a few pennies and a ruler.)
23. How many days would elapse before you had a total of over $1 billion? (*Hint*: Proceed by trial and error.)
24. Suppose that you could keep making a single stack of the pennies. After how many days would the stack be long enough to reach the nearest star (beyond the Sun), which is about 4.3 light-years (4.0×10^{13} km) away? (*Hint*: Proceed by trial and error.)

25–26: Bacteria in a Bottle Parable. Use the bacteria parable presented in the text.

25. How many bacteria are in the bottle at 11:50? What fraction of the bottle is full at that time?
26. How many bacteria are in the bottle at 11:15? What fraction of the bottle is full at that time?
27. **Knee-Deep in Bacteria.** The total surface area of Earth is about 5.1×10^{14} m^2. Assume that the bacteria continued their doublings for two hours (as discussed in the text), at which point they were distributed uniformly over Earth's surface. How deep would the bacterial layer be? Would it be knee-deep, more than knee-deep, or less than knee-deep? (*Hint*: Remember that a volume divided by an area gives a depth.)
28. **Bacterial Universe.** Suppose the bacteria in the parable continued to double their population every minute. How long would it take until their volume exceeded the total volume of the observable universe, which is about 10^{79} m^3? (*Hint*: Proceed by trial and error.)

FURTHER APPLICATIONS

29. **Human Doubling.** Human population in the year 2000 was about 6 billion. Suppose this population increases exponentially with a doubling time of 50 years.

a. Extend the following table, showing the population at 50-year intervals under this scenario, until you reach the year 3000. Use scientific notation, as shown.

Year	Population
2000	6×10^9
2050	$12 \times 10^9 = 1.2 \times 10^{10}$
2100	$24 \times 10^9 = 2.4 \times 10^{10}$
⋮	⋮

b. The total surface area of Earth is about 5.1×10^{14} m^2. Assuming that people could occupy all this area (in reality, most of it is ocean), approximately when would people be so crowded that every person would have only 1 m^2 of space?

c. Suppose that, when we take into account the area needed to grow food and to find other resources, each person actually requires about 10^4 m^2 of area to survive. About when would we reach this limit?