9. A rural population is falling at a rate of 20% per decade. If you wish to calculate its exact half-life, you should set the fractional growth rate per decade to

a. $r = 20$.

b. $r = 0.2$.

c. $r = -0.2$.

10. A new company's revenues are growing 15% per year. The doubling time for its revenues is

a. $\dfrac{\log_{10} 2}{\log_{10} 1.15}$ years.   b. $\dfrac{\log_{10} 2}{\log_{10} 0.85}$ years.

c. $\dfrac{\log_{10}(1 + 0.15)}{\log_{10} 2}$ years.

Exercises 8B

REVIEW QUESTIONS

1. What is a doubling time? Suppose a population has a doubling time of 25 years. By what factor will it grow in 25 years? in 50 years? in 100 years?

2. Given a doubling time, explain how you calculate the value of an exponentially growing quantity at any time t.

3. State the approximate doubling time formula and the conditions under which it works well. Give an example.

4. What is a half-life? Suppose a radioactive substance has a half-life of 1000 years. What fraction will be left after 1000 years? after 2000 years? after 4000 years?

5. Given a half-life, explain how you calculate the value of an exponentially decaying quantity at any time t.

6. State the approximate half-life formula and the conditions under which it works well. Give an example.

7. Briefly describe exact doubling time and half-life formulas. Explain all their terms.

8. Give an example in which it is important to use the exact doubling time or half-life formula, rather than the approximate formula. Explain why the approximate formula does not work well in this case.

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

9. Our town is growing with a doubling time of 25 years, so its population will triple in 50 years.

10. Our town is growing at a rate of 7% per year, so it will double in population about every 10 years.

11. A toxic chemical decays with a half-life of 10 years, so half of it will be gone 10 years from now and all the rest will be gone 20 years from now.

12. The half-life of plutonium-239 is about 24,000 years, so we can expect some of the plutonium produced in recent decades to still be around 100,000 years from now.

BASIC SKILLS & CONCEPTS

13–24: Logarithms. Refer to the Brief Review on p. 486. Determine whether each statement is true or false *without* doing any calculations. Explain your reasoning.

13. $10^{0.928}$ is between 1 and 10.

14. $10^{3.334}$ is between 500 and 1000.

15. $10^{-5.2}$ is between $-100{,}000$ and $-1{,}000{,}000$.

16. $10^{-2.67}$ is between 0.001 and 0.01.

17. $\log_{10} \pi$ is between 3 and 4.

18. $\log_{10} 96$ is between 3 and 4.

19. $\log_{10} 1{,}600{,}000$ is between 16 and 17.

20. $\log_{10}(8 \times 10^9)$ is between 9 and 10.

21. $\log_{10}\left(\frac{1}{4}\right)$ is between -1 and 0.

22. $\log_{10} 0.00045$ is between 5 and 6.

23. Using the approximation $\log_{10} 2 = 0.301$, find each of the following *without* a calculator.

a. $\log_{10} 8$ b. $\log_{10} 2000$ c. $\log_{10} 0.5$

d. $\log_{10} 64$ e. $\log_{10} 1/8$ f. $\log_{10} 0.2$

24. Using the approximation $\log_{10} 5 = 0.699$, find each of the following *without* a calculator.

a. $\log_{10} 50$ b. $\log_{10} 5000$ c. $\log_{10} 0.05$

d. $\log_{10} 25$ e. $\log_{10} 0.20$ f. $\log_{10} 0.04$

25–32: Doubling Time. Each exercise gives a doubling time for an exponentially growing quantity. Answer the questions that follow.

25. The doubling time of a population of fruit flies is 8 hours. By what factor does the population increase in 24 hours? in 1 week?

26. The doubling time of a bank account balance is 20 years. By what factor does it grow in 40 years? in 100 years?

27. The doubling time of a city's population is 17 years. How long does it take for the population to quadruple?

28. Prices are rising with a doubling time of 2 months. By what factor do prices increase in a year?

29. The initial population of a town is 10,000, and it grows with a doubling time of 10 years. What will the population be in 12 years? in 24 years?

30. The initial population of a town is 10,000, and it grows with a doubling time of 8 years. What will the population be in 12 years? in 24 years?

31. The number of cells in a tumor doubles every 2.5 months. If the tumor begins as a single cell, how many cells will there be after 2 years? after 4 years?

32. The number of cells in a tumor doubles every 6 months. If the tumor begins with a single cell, how many cells will there be after 6 years? after 10 years?

33–34: World Population. In 2009 world population was 6.8 billion. Use the given doubling time to predict the population in 2019, 2059, and 2109.

33. Assume a doubling time of 45 years.

34. Assume a doubling time of 60 years.

35. **Rabbits.** A community of rabbits begins with an initial population of 100 and grows 7% per month. Make a table, similar to Table 8.3, that shows the population for each of the next 15 months. Based on the table, find the doubling time of the population and briefly discuss how well the approximate doubling time formula works for this case.

36. **Mice.** A community of mice begins with an initial population of 1000 and grows 20% per month. Make a table, similar to Table 8.3, that shows the population for each of the next 15 months. Based on the table, find the doubling time of the population and briefly discuss how well the approximate doubling time formula works for this case.

37–40: Doubling Time Formula. Use the approximate doubling time formula (rule of 70). Discuss whether the formula is valid for the case described.

37. The Consumer Price Index is increasing at a rate of 4% per year. What is its doubling time? By what factor will prices increase in 3 years?

38. A city's population is growing at a rate of 3.5% per year. What is its doubling time? By what factor will the population increase in 50 years?

39. Prices are rising at a rate of 0.3% per month. What is their doubling time? By what factor will prices increase in 1 year? in 8 years?

40. Oil consumption is increasing at a rate of 2.2% per year. What is its doubling time? By what factor will oil consumption increase in a decade?

41–48: Half-Life. Each exercise gives a half-life for an exponentially decaying quantity. Answer the questions that follow.

41. The half-life of a radioactive substance is 50 years. If you start with some amount of this substance, what fraction will remain in 100 years? in 200 years?

42. The half-life of a radioactive substance is 400 years. If you start with some amount of this substance, what fraction will remain in 120 years? in 2500 years?

43. The half-life of a drug in the bloodstream is 18 hours. What fraction of the original drug dose remains in 24 hours? in 48 hours?

44. The half-life of a drug in the bloodstream is 4 hours. What fraction of the original drug dose remains in 24 hours? in 48 hours?

45. The current population of a threatened animal species is 1 million, but it is declining with a half-life of 20 years. How many animals will be left in 30 years? in 70 years?

46. The current population of a threatened animal species is 1 million, but it is declining with a half-life of 25 years. How many animals will be left in 20 years? in 40 years?

47. Cobalt-56 has a half-life of 77 days. If you start with 1 kilogram of cobalt-56, how much will remain after 150 days? after 300 days?

48. Radium-226 is a metal with a half-life of 1600 years. If you start with 1 kilogram of radium-226, how much will remain after 1000 years? after 10,000 years?

49–52: Half-Life Formula. Use the approximate half-life formula. Discuss whether the formula is valid for the case described.

49. Urban encroachment is causing the area of a forest to decline at a rate of 7% per year. What is the half-life of the forest? What fraction of the forest will remain in 30 years?

50. A clean-up project is reducing the concentration of a pollutant in the water supply, with a 3.5% decrease per week. What is the half-life of the concentration of the pollutant? What fraction of the original amount of the pollutant will remain when the project ends after 1 year (52 weeks)?

51. Poaching is causing a population of elephants to decline by 8% per year. What is the half-life for the population? If there are 10,000 elephants today, how many will remain in 50 years?

52. The production of a gold mine is declining by 6% per year. What is the half-life for the production decline? If its current annual production is 5000 kilograms, what will its production be in 15 years?