Exercises 9A

REVIEW QUESTIONS

1. What is a mathematical model? Explain this statement: *A model's predictions can be only as good as the data and the assumptions from which the model is built.*
2. What is a function? How do you decide which variable is the independent variable and which is the dependent variable?
3. What are the three basic ways to represent a function?
4. Define *domain* and *range*, and explain how to determine them for a particular function.

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

5. Scientists at the National Center for Atmospheric Research use mathematical models to learn about the Earth's climate.
6. The demand for concert tickets is a function of their price.
7. I graphed a function showing how my heart rate depends on my running speed. The domain was heart rates from 60 to 180 beats per minute.
8. My mathematical model fits the data perfectly, so I can be confident it will work equally well in any new situations we encounter.

BASIC SKILLS & CONCEPTS

9–10: Coordinate Plane Review. Use the skills covered in the Brief Review on p. 515.

9. Draw a set of axes in the coordinate plane. Plot and label the following points: $(0, 1)$, $(-2, 0)$, $(1, 5)$, $(-3, 4)$, $(5, -2)$, $(-6, -3)$.
10. Draw a set of axes in the coordinate plane. Plot and label the following points: $(0, -1)$,$(2, -1)$,$(6, 5)$,$(3, -4)$,$(-5, -2)$, $(-6, 2)$.

11–18: Related Quantities. Write a short statement that expresses a possible relationship between the variables.

Example: (age, shoe size).

Solution: As a child ages, shoe size increases. Once the child is full-grown, shoe size remains constant.

11. (*volume of gas tank, cost to fill the tank*)
12. (*time, price of a Ford station wagon*), where time represents years from 1970 to 2015
13. (*time in seconds after jumping from an airplane, speed of skydiver*)
14. (*unit price of DVDs, the number of DVDs sold in a week*)
15. (*travel time from Denver to Chicago, average speed of car*)
16. (*rate of pedaling, speed of bicycle*)
17. (*gas mileage of car, cost of driving 500 miles*)
18. (*annual percentage rate (APR), balance in savings account after 10 years*)
19. **Pressure Function.** Study Figure 9.6.

    a. Use the graph to estimate the pressure at altitudes of 6000 feet, 18,000 feet, and 29,000 feet.

    b. Use the graph to estimate the altitudes at which the pressure is 23, 19, and 13 inches of mercury.

    c. Estimating beyond the boundaries of the graph, at what altitude do you think the atmospheric pressure reaches 5 inches of mercury? Is there an altitude at which the pressure is exactly zero? Explain your reasoning.
20. **Daylight Function.** Study Figure 9.7, which applies to 40°N latitude.

    a. Use the graph to estimate the number of hours of daylight on April 1 (the 91st day of the year) and October 31 (the 304th day of the year).

    b. Use the graph to estimate the dates on which there are 13 hours of daylight.

    c. Use the graph to estimate the dates on which there are 10.5 hours of daylight.

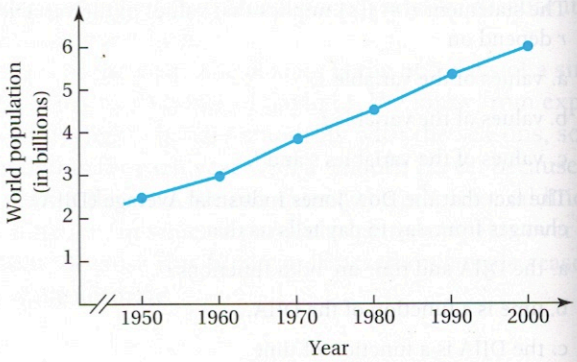
    d. The graph in Figure 9.7 is valid at 40°N latitude. How do you think the graph would be different at 20°N latitude, 60°N latitude, and 40°S latitude? Why?

21–22: Functions from Graphs. Consider the graphs of the following functions.

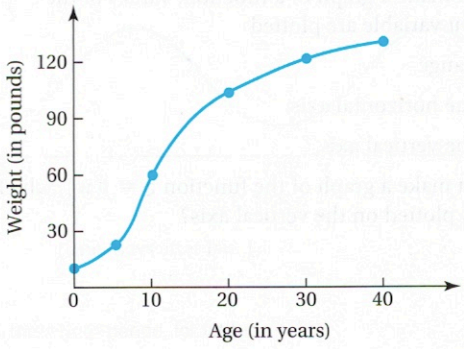
a. Identify the independent and dependent variables, and describe the domain and range.

b. Describe the function in words.

21.



22.



23–26: Functions from Data Tables. Each of the following data tables represents a function.

a. Identify the independent and dependent variables, and describe the domain and range.

b. Make a clear graph of the function. Explain how you decide on the shape of the curve used to fill in the gaps between the data points.

c. Describe the function in words.

23.

Date	Average High Temperature
Jan. 1	42°F
Feb. 1	38°F
Mar. 1	48°F
Apr. 1	58°F
May 1	69°F
June 1	76°F
July 1	85°F
Aug. 1	83°F
Sep. 1	80°F
Oct. 1	69°F
Nov. 1	55°F
Dec. 1	48°F
Dec. 31	44°F

24.

Altitude (ft)	Boiling Point of Water (°F)
0	212.0
1000	210.2
2000	208.4
3000	206.6
4000	204.8
5000	203.0
6000	201.0
7000	199.3
8000	195.5
9000	193.6

25. Notice that the intervals between data points are not the same in all cases.

Year	Tobacco Produced (billions of lb)	Year	Tobacco Produced (billions of lb)
1975	2.2	1986	1.2
1980	1.8	1987	1.2
1982	2.0	1988	1.4
1984	1.7	1989	1.4
1985	1.5	1990	1.6

26.

Year	Projected U.S. Population (millions)
2010	309
2015	323
2020	336
2025	351
2030	364
2035	379
2040	393

FURTHER APPLICATIONS

27–38: Rough Sketches of Functions. For each function, use your intuition or additional research, if necessary, to do the following.

a. Describe an appropriate domain and range for the function.

b. Make a rough sketch of a graph of the function. Explain the assumptions that go into your sketch.

c. Briefly discuss the validity of your graph as a model of the true function.

27. (*altitude, temperature*) when climbing a mountain

28. (*day of year, high temperature*) over a two-year period for the town in which you are living

29. (*blood alcohol content, reflex time*) for a single person

30. (*number of pages in a book, time to read the book*) for a single person

31. (*time of day, traffic flow*) at a busy intersection over a full day

32. (*price of gasoline, number of tourists in Yellowstone*)

33. (*time, world record in the 100-meter dash*) between 1970 and 2004

34. (*minutes after lighting, length of candle*)

35. (*time, population of China*), where time is measured in years after 1900

36. (*time of day, elevation of tide*) at a particular seaside port over two days

37. (*angle of cannon, horizontal distance traveled by cannonball*)

38. (*weight of car, average gas mileage*)

WEB PROJECTS

39. **Daylight Hours.** Investigate Web sites that give the length of day (hours of daylight) on various days throughout the year for different latitudes (a table of sunrise and sunset times would also work). Make graphs similar to Figure 9.7 showing the variation of hours of daylight over a year for several different latitudes.

40. **Variable Tables.** Find data on the Web for two variables that are clearly related in some way. Make a table (between 10 and 20 entries) of data values. Graph the data and describe in words the function that relates the variables. *Hint:* Some possible variable pairs are (*time, population of a city*), (*team batting average, average team salary*) for major league baseball teams, and (*blood alcohol content, reaction time*) for a study of effects of alcohol.

IN YOUR WORLD

41. **Everyday Models.** Describe three different models (mathematical or otherwise) that you use or encounter frequently in everyday life. What is the underlying "reality" that those models represent? What simplifications are made in constructing those models?

42. **Functions and Variables.** Identify three different variables in recent news stories. For each variable, specify another related variable and then write a paragraph that describes a function relating the two variables. At least one of your three functions should *not* use time as the independent variable.