4. Suppose that Figure 9.11 is an accurate representation of elevation changes for the first 3 miles of a much longer trail. If you have no data other than those shown, what should you predict for the elevation at mile 5 of the trail?

   a. The elevation is $(8000 + 5 \times 650)$ feet.

   b. You should not make a prediction, because the elevation at 5 miles must be higher than the 10,000-foot maximum shown on the graph.

   c. You should not make a prediction, because mile 5 of the trail is not within the domain of the function shown.

5. Which town would have the steepest slope on a graph showing its population as a function of time?

   a. a town growing at a constant rate of 50 people per year

   b. a town growing at a constant rate of 75 people per year

   c. a town growing at a constant rate of 100 people per year

6. Consider the function $price = \$100 - (\$3/yr) \times time$. The initial value of this function is

   a. $100.   b. $3.   c. $0.

7. Consider the demand function given in Example 6, which is $d = 100 - 10p$. A graph of this function would

   a. slope upward, starting from a price of $100.

   b. slope upward, starting from a price of $0.

   c. slope downward, starting from a price of $100.

8. A line intersects the y-axis at a value of $y = 7$ and has a slope of -2. The equation of this line is

   a. $y = -2x + 7$.   b. $y = 7x - 2$.   c. $y = 2x - 7$.

9. Consider a line with equation $y = 12x - 3$. Which of the following lines has the same slope but a different y-intercept?

   a. $y = \frac{12}{3}x - \frac{3}{3}$ b. $y = 12x + 3$ c. $y = -12x - 3$

10. Charlie picks apples in the orchard at a constant rate. By 9:00 a.m. he has picked 150 apples, and by 11:00 a.m. he has picked 550 apples. If we use A for the number of apples and t for time measured in hours since 9:00 a.m., which of the following functions describes his harvesting?

    a. $A = 150t + 2$ b. $A = 550t + 150$ c. $A = 200t + 150$

Exercises 9B

REVIEW QUESTIONS

1. What does it mean to say that a function is linear?

2. Define *rate of change*, and describe how a rate of change is stated in words (that is, using "with respect to").

3. How is the rate of change of a linear function related to the slope of its graph?

4. How do you find the change in the dependent variable, given a change in the independent variable? Give an example.

5. Describe the general equation for a linear function. How is it related to the standard algebraic form $y = mx + b$?

6. Describe the process of creating an equation for a linear function from two data points. How are such models useful?

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.
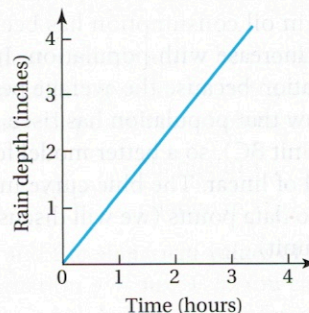
7. When I graphed the linear function, it turned out to be a wavy curve.

8. I graphed two linear functions, and the one with the greater rate of change had the greater slope.

9. My freeway speed is the rate of change in my distance with respect to time.

10. It's possible to make a linear model from any two data points, but there's no guarantee that the model will fit other data points.
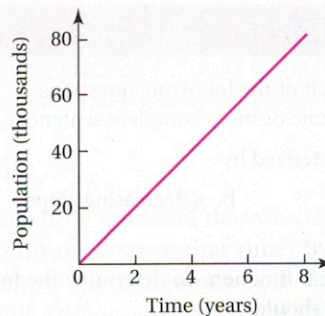
BASIC SKILLS & CONCEPTS

11–16: Linear Functions. Consider the following graphs.

a. In words, describe the function shown on the graph.

b. Find the slope of the graph and express it as a rate of change (be sure to include units).

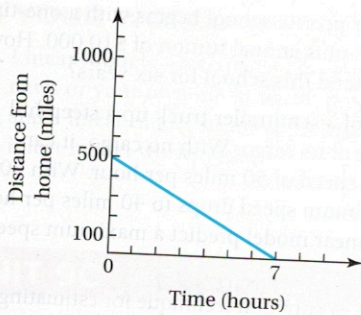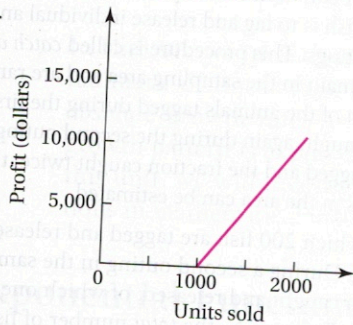c. Briefly discuss the conditions under which a linear function is a realistic model for the given situation.
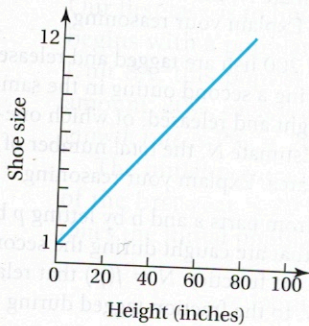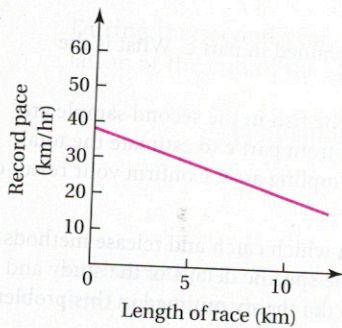
11.

12.

13.

(Graph: y-axis "Distance from home (miles)" with values 100, 500, 1000; x-axis "Time (hours)" from 0 to 7)

14.

(Graph: y-axis "Profit (dollars)" with values 5,000, 10,000, 15,000; x-axis "Units sold" with values 1000, 2000)

15.

(Graph: y-axis "Shoe size" with values 1, 12; x-axis "Height (inches)" with values 20, 40, 60, 80, 100)

16.

(Graph: y-axis "Record pace (km/hr)" with values 10, 20, 30, 40, 50, 60; x-axis "Length of race (km)" with values 5, 10)

17–22: Rate of Change Rule. The following situations involve a rate of change that is constant. Write a statement that describes how one variable varies with respect to the other, give the rate of change numerically (with units), and use the rate of change rule to answer any questions.

Example: Every week your fingernails grow 5 millimeters. How much will your fingernails grow in 2.5 weeks?

Solution: The *length of your fingernails* varies with respect to *time,* with a rate of change of 5 mm/wk. In 2.5 weeks, your fingernails will grow 5 mm/wk × 2.5 wk = 12.5 millimeters.

17. The water depth in a lake decreases at a rate of 2 inches per day because of evaporation. How much does the water depth change in 8 days? in 15 days?

18. You drive along the highway at a constant speed of 65 miles per hour. How far do you travel in 3.5 hours? in 6.8 hours?

19. A 1-degree change (increase or decrease) on the Celsius temperature scale is equivalent to a 9/5-degree change on the Fahrenheit temperature scale. How much does the Fahrenheit temperature increase if the Celsius temperature increases 5 degrees? How much does the Fahrenheit temperature decrease if the Celsius temperatue decreases 25 degrees?

20. A gas station owner finds that for every penny increase in the price of gasoline, she sells 120 fewer gallons of gas per week. How much more or less gas will she sell if she raises the price by 10 cents per gallon? if she decreases the price by 5 cents per gallon?

21. Snow accumulates during a storm at a constant rate of 3.5 inches per hour. How much snow accumulates in the first 6.3 hours? in the first 9.8 hours?

22. According to one formula, your maximum heart rate (in beats per minute) is 220 minus your age (in years). How much does your maximum heart rate change from age 25 to age 40? What is your maximum heart rate at age 70?

23–28: Linear Equations. The following situations can be modeled by linear functions. In each case, write an equation for the linear function and use it to answer the given question. Be sure you clearly identify the independent and dependent variables. Then briefly discuss whether a linear model is reasonable for the situation described.

23. The price of a particular model car is $15,000 today and rises with time at a constant rate of $800 per year. How much will a new car cost in 3.5 years?

24. The world record time in the 100-meter butterfly (swimming) was 50.40 seconds in 2006. Assume that the record falls at a constant rate of 0.05 second per year. What does the model predict for the record in 2020?

25. A snowplow has a maximum speed of 40 miles per hour on a dry highway. Its maximum speed decreases by 1.1 miles per hour for every inch of snow on the highway. According to this model, at what snow depth will the plow be unable to move?

26. The cost of leasing a car is $800 for a down payment and processing fee plus $240 per month. For how many months can you lease a car with $3680?

27. You can rent time on computers at the local copy center for an $8 setup charge and an additional $1.50 for every 5 minutes. How much time can you rent for $25?

28. In 1980, the population of Boom Town began increasing at a rate of 200 people per year. The 1980 population was 2000 people. What is your projection for the population in the year 2010?

29–34: Equations from Two Data Points. Create the required linear function and use it to answer the following questions.

29. Suppose your pet dog weighed 2.5 pounds at birth and weighed 15 pounds after one year. Based on these two data points, find a linear function that describes how weight varies with age. Use this function to predict your dog's weight at 5 and 10 years of age. Comment on the validity of the model.

30. You can purchase a motorcycle for $6500 or lease it for a down payment of $200 and $150 per month. Find a function that describes how the cost of the lease depends on time. How long can you lease the motorcycle before you've paid more than its purchase price?

31. A Campus Republicans fundraiser offers raffle tickets for $10 each. The prize for the raffle is a $350 television set, which must be purchased with proceeds from the ticket sales. Find a function that gives the profit/loss for the raffle as it varies with the number of tickets sold. How many tickets must be sold for the raffle sales to equal the cost of the prize?

32. The Campus Democrats plan to pay a visitor $100 to speak at a fundraiser. Tickets will be sold for $4 apiece. Find a function that gives the profit/loss for the event as it varies with the number of tickets sold. How many people must attend the event for the club to break even?

33. A $1200 washing machine in a laundromat is depreciated for tax purposes at a rate of $75 per year. Find a function for the depreciated value of the washing machine as it varies with time. When does the depreciated value reach $0?

34. A mining company can extract 2000 tons of gold ore per day with a purity of 3 ounces of gold per ton. The cost of extraction is $1000 per ton. If p is the price of gold in dollars per ounce, find a function that gives the daily profit/loss of the mine as it varies with the price of gold. What is the minimum price of gold that makes the mine profitable?

FURTHER APPLICATIONS

35–42: Algebraic Linear Equations. For the following functions, find the slope of the graph and the y-intercept. Then sketch the graph for values of x between -10 and 10.

35. $y = 2x + 6$

36. $y = -3x + 3$

37. $y = -5x - 5$

38. $y = 4x + 1$

39. $y = 3x - 6$

40. $y = -2x + 5$

41. $y = -x + 4$

42. $y = 2x + 4$

43–48: Linear Graphs. The following situations can be modeled by linear functions. In each case, draw a graph of the function and use the graph to answer the given question. Be sure you clearly identify the independent and dependent variables. Then briefly discuss whether a linear model is reasonable for the situation described.

43. A group of climbers begin climbing at an elevation of 6500 feet and ascend at a steady rate of 600 vertical feet per hour. What is their elevation after 3.5 hours?

44. The diameter of a tree increases by 0.2 inch with each passing year. When you started observing the tree, its diameter was 4 inches. Estimate the time at which the tree started growing.

45. The cost of publishing a poster is $2000 for setting up the printing equipment, plus $3 per poster printed. What is the total cost to produce 2000 posters?

46. The amount of sugar in a fermenting batch of beer decreases with time at a rate of 0.1 gram per day, starting from an initial amount of 5 grams. When is the sugar gone?

47. The cost of a particular private school begins with a one-time initiation fee of $2000, plus annual tuition of $10,000. How much will it cost to attend this school for six years?

48. The maximum speed of a semitrailer truck up a steep hill varies with the weight of its cargo. With no cargo, it can maintain a maximum speed of 50 miles per hour. With 20 tons of cargo, its maximum speed drops to 40 miles per hour. At what load does a linear model predict a maximum speed of 0 miles per hour?

49. **Wildlife Management.** A common technique for estimating populations of birds or fish is to tag and release individual animals in two different outings. This procedure is called *catch and release*. If the wildlife remain in the sampling area and are randomly caught, a fraction of the animals tagged during the first outing are likely to be caught again during the second outing. Based on the number tagged and the fraction caught twice, the total number of animals in the area can be estimated.

   a. Consider a case in which 200 fish are tagged and released during the first outing. During a second outing in the same area, 200 fish are again caught and released, of which one-half are already tagged. Estimate N, the *total* number of fish in the entire sampling area. Explain your reasoning.

   b. Consider a case in which 200 fish are tagged and released during the first outing. During a second outing in the same area, 200 fish are again caught and released, of which one-fourth are already tagged. Estimate N, the *total* number of fish in the entire sampling area. Explain your reasoning.

   c. Generalize your results from parts a and b by letting p be the fraction of tagged fish that are caught during the second outing. Find a formula for the function $N = f(p)$ that relates the total number of fish, N, to the fraction tagged during the second outing, p.

   d. Graph the function obtained in part c. What is the domain? Explain.

   e. Suppose that 15% of the fish in the second sample are tagged. Use the formula from part c to estimate the total number of fish in the sampling area. Confirm your result on your graph.

   f. Locate a real study in which catch and release methods were used. Report on the specific details of the study and how closely it followed the theory outlined in this problem.

WEB PROJECTS

50. **Alcohol Metabolism.** Most drugs are eliminated from the blood through an exponential decay process with a constant half-life (see Unit 9C). Alcohol is an exception in that it is metabolized through a linear decay process. Find data showing how the blood alcohol content (BAC) decreases over time, and use the data to develop a linear model. Discuss the validity of the model. What assumptions (for example, gender, weight, number of drinks) were used in creating the model?

51. **Property Depreciation.** Go to the IRS Web site and examine the rules for depreciation of some type of property, such as a rental property or a piece of business equipment. Make a linear model that describes the depreciation function.