QUICK QUIZ

Choose the best answer to each of the following questions. Explain your reasoning with one or more complete sentences.

1. A city's population starts at 100,000 people and grows 3% per year for 7 years. In the general exponential equation $Q = Q_0 \times (1 + r)^t$, what is Q_0?

   a. 100,000 b. 3 c. 7

2. A city's population starts at 100,000 people and grows 3% per year for 7 years. In the general exponential equation $Q = Q_0 \times (1 + r)^t$, what is r?

   a. 3 b. 0.03 c. 7

3. India's 2009 population was estimated to be 1.15 billion, with a growth rate of 1.6% per year. If the growth rate remains steady, its 2020 population will be

   a. 1.15 billion $\times\ 1.016^{11}$. b. 1.15 billion $\times\ 11^{1.6}$.

   c. $2020 \times (1.15 \text{ billion})^{0.016}$.

4. Suppose that inflation is causing the value of a dollar to decrease at a rate of 4.5% per year. To use the general exponential equation to find the value of the dollar at some future time compared to its present value, you would set r to

   a. 4.5. b. 0.045. c. −0.045.

5. Figure 9.18b shows the graph of an exponentially decaying quantity. In theory, how many half-lives would it take for the value of Q to reach zero?

   a. 6 b. 12 c. The value of Q never reaches zero.

6. Polly received a large dose of an antibiotic, and you want to know how much antibiotic remains in her body after 3 days. Which two pieces of information are sufficient for you to calculate the answer?

   a. her body weight and the rate at which the antibiotic is metabolized

   b. the amount of the initial dose and the half-life of the antibiotic in the bloodstream

   c. the rate at which the antibiotic is metabolized and the half-life of the antibiotic in the bloodstream

7. The half-life of carbon-14 is 5700 years, and carbon-14 is incorporated into the bones of a living organism only while it is alive. Suppose you have found a human bone at an archaeological site and you want to use carbon-14 to determine how long ago the person died. Which of the following additional pieces of information would allow you to do the calculation?

   a. only the amount of carbon-14 in the bone today

   b. both the amount of carbon-14 in the bone today and the rate at which carbon-14 decays

   c. both the amount of carbon-14 in the bone today and the amount it contained at the time the person died

8. Radioactive uranium-235 has a half-life of about 700 million years. Suppose you find a rock and chemical analysis tells you that only 1/16 of the rock's original uranium-235 remains. How old is the rock?

   a. 1.4 billion years old

   b. 2.1 billion years old

   c. 2.8 billion years old

9. Suppose you find a rock that contains 1/25 of its original uranium-235, which has a half-life of about 700 million years. The age of this rock is

   a. $t = 700 \text{ millon years} \times \dfrac{\log_{10} \frac{1}{25}}{\log_{10} \frac{1}{2}}$.

   b. $t = \dfrac{1}{25} \times 700 \text{ millon years}$.

   c. $t = 700 \text{ millon years} \times \dfrac{1}{25} \times \log_{10} \dfrac{1}{2}$.

10. Compare the first two forms of the exponential function in the box on page 540. Given that these two forms are equivalent, what can you conclude?

    a. $(1 + r)^t = 2^{t/T_{\text{double}}}$

    b. $r = 1 - 2^{t/T_{\text{double}}}$

    c. $Q = Q_0$ whenever $t = T_{\text{double}}$

Exercises 9C

REVIEW QUESTIONS

1. Describe the meanings of all the variables in the exponential function. Explain how the function is used for exponential growth and decay.
2. Briefly explain how to find the doubling time and half-life from the exponential equation.
3. Describe how you can graph an exponential function with the help of the doubling time or half-life. What is the general shape of an exponential growth function? What is the general shape of an exponential decay function?
4. Describe the meaning of each of the three forms of the exponential function given in this chapter. Under what circumstance is each form useful?
5. Briefly describe how exponential functions are useful for modeling inflation, environmental and resource issues, physiological processes, and radioactive decay.

6. Briefly describe the process of radiometric dating. What makes it difficult? How can the difficulty be alleviated?

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

7. After 100 years, a population growing at a rate of 2% per year will have grown by twice as many people as a population growing at a rate of 1% per year.
8. When I used the exponential function to model the decay of the medicine in my bloodstream, the growth rate r was negative.
9. We can use the fact that radioactive materials decay exponentially to determine the ages of ancient bones from archaeological sites.
10. I used the exponential function to figure how much money I'd have in a bank account that earns compound interest.

BASIC SKILLS & CONCEPTS

11–26: Review of Logarithms. Use the skills covered in the Brief Review on p. 537 to solve the following equations for the unknown quantity x.

11. $2^x = 128$
12. $10^x = 23$
13. $3^x = 99$
14. $5^{2x} = 240$
15. $7^{3x} = 623$
16. $3 \times 4^x = 180$
17. $9^x = 1748$
18. $3^{x/4} = 444$
19. $\log_{10}x = 4$
20. $\log_{10}x = -3$
21. $\log_{10}x = 3.5$
22. $\log_{10}x = -2.2$
23. $3\log_{10}x = 4.2$
24. $\log_{10}(3x) = 5.1$
25. $\log_{10}(4 + x) = 1.1$
26. $4\log_{10}(4x) = 4$

27–34. Exponential Growth and Decay Laws. Consider the following cases of exponential growth and decay.

a. Create an exponential function of the form $Q = Q_0 \times (1 + r)^t$ (where $r > 0$ for growth and $r < 0$ for decay) to model the situation described. Be sure to clearly identify both variables in your function.

b. Create a table showing the value of the quantity Q for the first 10 units of time (either years, months, weeks, or hours) of growth or decay.

c. Make a graph of the exponential function.

27. The population of a town with an initial population of 60,000 grows at a rate of 2.5% per year.
28. The number of restaurants in a city that had 800 restaurants in 2001 is increasing at a rate of 3% per year.
29. A privately owned forest that had 1 million acres of old growth is being clear cut at a rate of 7% per year.

30. A town with a population of 10,000 is losing residents at a rate of 0.3% per month because of a poor economy.
31. The average price of a home in a town was $175,000 in 2007 but home prices are rising by 5% per year.
32. A certain drug breaks down in the human body at a rate of 15% per hour. The initial amount of the drug in the bloodstream is 8 milligrams.
33. Your starting salary at a new job is $2000 per month, and you get annual raises of 5% per year.
34. You hid 100,000 rubles in a mattress at the end of 1991, when they had a value of $10,000. However, the value of the ruble against the dollar then fell 50% per year.

35–36: Annual vs. Monthly Inflation. Answer the following questions about monthly and annual inflation rates.

35. If prices increase at a monthly rate of 1.5%, by what percentage do they increase in a year?
36. If the price of gold decreases at a monthly rate of 1%, by what percentage does it decrease in a year?
37. **Hyperinflation in Brazil.** During the 1970s and 1980s, Brazil underwent tremendous *hyperinflation*—periods of extraordinarily large inflation in prices. During the worst periods, prices rose as rapidly as 80% per month. At this rate, by what percentage would prices have risen in 1 year? in 1 day?
38. **Hyperinflation in Bosnia.** During the war in Bosnia, inflation sometimes drove prices up at a rate of 90% per month. At this rate, by what percentage would prices rise in 1 year? in 1 day?
39. **Extinction by Poaching.** Suppose that poaching reduces the population of an endangered animal by 8% per year. Further suppose that when the population of this animal falls below 30, its extinction is inevitable (owing to the lack of reproductive options without severe in-breeding). If the current population of the animal is 1500, when will it face extinction? Comment on the validity of the exponential model.

40. **World Oil Production.** Annual world oil production was 518 million tons in 1950. Production increased at a rate of 7% per year between 1950 and 1972, but the rate of growth then slowed. World oil production reached approximately 3.9 billion tons per year in 2008.

a. What was world oil production in 1972?

b. Using the result of part a, determine how much oil would have been produced in 2008 if growth in production had continued at a rate of 7% between 1972 and 2008. Compare this result to the actual 2008 figure given above.

c. Using the result of part a, determine how much oil would have been produced in 2008 if growth in production had proceeded at a rate of 3% between 1972 and 2008. Compare this result to the actual 2008 figure given above.

d. By trial and error, estimate the annual growth rate in world oil production between 1972 and 2008 with an exponential function.

41. **Valium Metabolism.** The drug Valium is eliminated from the bloodstream exponentially with a half-life of 36 hours. Suppose that a patient receives an initial dose of 50 milligrams of Valium at midnight.

a. How much Valium is in the patient's blood at noon the next day?

b. Estimate when the Valium concentration will reach 10% of its initial level.

42. **Aspirin Metabolism.** Assume that for the average individual, aspirin has a half-life of 8 hours in the bloodstream. At 12:00 noon, you take a 300-milligram dose of aspirin.

a. How much aspirin will be in your blood at 6:00 p.m. the same day? at midnight? at 12:00 noon the next day?

b. Estimate when the amount of aspirin will decay to 5% of its original amount.

43–44: Radiometric Dating. Use the radiometric dating formula to answer the following questions.

43. Uranium-238 has a half-life of 4.5 billion years.

a. You find a rock containing a mixture of uranium-238 and lead. You determine that 65% of the original uranium-238 remains; the other 35% decayed into lead. How old is the rock?

b. Analysis of another rock shows that it contains 45% of its original uranium-238; the other 55% decayed into lead. How old is the rock?

44. The half-life of carbon-14 is about 5700 years.

a. You find a piece of cloth painted with organic dyes. By analyzing the dye in the cloth, you find that only 63% of the carbon-14 originally in the dye remains. When was the cloth painted?

b. A well-preserved piece of wood found at an archaeological site has 12.3% of the carbon-14 that it must have had when it was alive. Estimate when the wood was cut.

c. Is carbon-14 useful for establishing the age of the Earth? Why or why not?

FURTHER APPLICATIONS

45. **Radioactive Waste.** A toxic radioactive substance with a density of 3 milligrams per square centimeter is detected in the ventilating ducts of a nuclear processing building that was used 55 years ago. If the half-life of the substance is 20 years, what was the density of the substance when it was deposited 55 years ago?

46. **Metropolitan Population Growth.** A small city had a population of 110,000 in 2010. Concerned about rapid growth, the residents passed a growth control ordinance limiting population growth to 2% each year. If the population grows at this 2% annual rate, what will the population be in 2020? What is the maximum growth rate cap that will prevent the population from reaching 150,000 in 2025?

47. **Rising Costs.** Between 2005 and 2010 the average rate of inflation was about 3.2% per year (as measured by the Consumer Price Index). If a cart of groceries cost $150 in 2005, what did it cost in 2010?

48. **Periodic Drug Doses.** It is common to take a drug (such as aspirin or an antibiotic) repeatedly at fixed time intervals. Suppose that an antibiotic has a half-life of 8 hours and a 100-milligram dose is taken every 8 hours.

a. Write an exponential function that represents the decay of the antibiotic from the moment of the first dose to just *prior* to the next dose (i.e., 8 hours after the first dose). How much antibiotic is in the bloodstream just *prior* to this next dose? How much antibiotic is in the bloodstream just *after* this next dose?

b. Following a procedure similar to that in part a, calculate the amounts of antibiotic in the bloodstream just prior to and just after the doses at 16 hours, 24 hours, and 32 hours.

c. Make a graph of the amount of antibiotic in the bloodstream for the first 32 hours after the first dose of the drug. What do you predict will happen to the amount of drug if the doses every 8 hours continue for several days or weeks? Explain.

d. Consult a pharmacist (or read the fine print on the information sheet enclosed with many medicines) to find the half-lives of some common drugs. Create a model for the metabolism of one drug using the above procedure.

49. **Increasing Atmospheric Carbon Dioxide.** Between 1860 and 2010, carbon dioxide (CO_2) concentration in the atmosphere rose from roughly 290 parts per million to 380 parts per million. Assume that this growth can be modeled with an exponential function of the form $Q = Q_0 \times (1 + r)^t$.

a. By experimenting with various values of the fractional growth rate r, find an exponential function that fits the given data for 1860 and 2010.

b. Use this exponential model to predict when the CO_2 concentration will be double its 1860 level.

c. Many nations have proposed or set precise goals for reductions in carbon dioxide emissions, Research the current goals of the United States. Are goals proposed in the past being met?