

§ 2.3 MEASURES OF VARIABILITY

$$\underline{2.13} \quad (a) \quad \bar{x} = \frac{2 + 1 + 1 + 3 + 5}{5} = \frac{12}{5} = 2.4$$

$$(b) \quad s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

$$= \frac{(2-2.4)^2 + (1-2.4)^2 + (1-2.4)^2 + (3-2.4)^2 + (5-2.4)^2}{4}$$

$$= \frac{0.16 + 1.96 + 1.96 + 0.36 + 6.76}{4}$$

$$= 2.8$$

$$(c) \quad s = \sqrt{s^2} = \sqrt{2.8} \approx 1.673$$

$$(d) \quad s^2 = \frac{\sum_{i=1}^n x_i^2 - \frac{1}{n} \left(\sum_{i=1}^n x_i \right)^2}{n-1}$$

$$= \frac{(2^2 + 1^2 + 1^2 + 3^2 + 5^2) - \frac{1}{5} (2 + 1 + 1 + 3 + 5)^2}{4}$$

$$= \frac{40 - \frac{1}{5} (144)}{4} = 2.8 \quad \checkmark$$

$$\underline{2.16} \quad (a) \text{ RANGE} = 6 - 1 = 5$$

$$(b) \bar{x} = \frac{31}{8} = 3.875$$

$$(c) s^2 = \frac{\sum_{i=1}^8 x_i^2 - \frac{1}{8} \left(\sum_{i=1}^8 x_i \right)^2}{7}$$

$$= \frac{3^2 + 1^2 + 5^2 + 6^2 + 4^2 + 4^2 + 3^2 + 5^2 - \frac{1}{8} (3+1+5+6+4+4+3+5)^2}{7}$$

$$= \frac{137 - \frac{1}{8} (31)^2}{7} = 2.41\dots \rightarrow s = \sqrt{2.41\dots} \\ = 1.55\dots$$

(d) THE RANGE (5) IS APPROXIMATELY 3 - 4 STANDARD DEVIATIONS.

$$\left(\frac{\text{RANGE}}{s} = \frac{5}{1.55} \approx 3.226\dots \right)$$

$$\underline{2.17} \quad (a) \text{ RANGE} = 2.39 - 1.28 = 1.11$$

$$(b) s^2 = \frac{\sum x_i^2 - \frac{1}{n} (\sum x_i)^2}{n-1}$$

$$= \frac{1.28^2 + 2.39^2 + 1.5^2 + 1.88^2 + 1.51^2 - \frac{1}{5} (8.56)^2}{4}$$

$$\boxed{s^2 = 0.19007}, \quad \boxed{s = \sqrt{0.19007} \approx 0.436}$$

(c) RANGE IS APPROXIMATELY 2 - 3 STANDARDS DEVIATIONS

$$\left(\frac{\text{RANGE}}{s} = \frac{1.11}{0.436} \approx 2.55 \right)$$