Name: *Answer Key* **5/3/2017**

Math 173 Introduction to Probability and Statistics **Quiz 4**

Please box your final answers. Calculators are allowed, but not required. Answers may be left as fractions and/or expressions that may contain square-root ($\sqrt{\cdot}$), factorial (!), permutation (P_r^n), and combination (C_r^n) notation.

1. A normally distributed random variable x has mean $\mu = 72$ and a standard deviation $\sigma = 4$. Find the following probabilities.

   (a) (4 points) $P(x \leq 70)$

   $z = \frac{x-\mu}{\sigma} = \frac{70-72}{4} = -.5$

   $P(x \leq 70) = P(z \leq -.5)$

   $= \boxed{.3085}$

   (b) (4 points) $P(x \geq 80)$

   $z = \frac{80-72}{4} = 2$

   $P(x \geq 2) = P(z \geq 2) = 1 - P(z \leq 2)$

   $= 1 - .9772$

   $= \boxed{.0228}$

   (c) (4 points) $P(70 < x < 80)$

   $P(70 < x < 80) = P(x \leq 80) - P(x \leq 70) = .9772 - .3085$

   $= \boxed{.6687}$

2. (8 points) Suppose IQ scores are normally distributed with mean $\mu = 100$ and standard deviation $\sigma = 15$. How high must a person score in order to be in the 98^{th} percentile (that is, in order to score higher than 98% of the population)?

FIRST FIND z_0 SUCH THAT $P(z \leq z_0) = .98$

FROM TABLE 3, $z_0 = 2.05$ OR 2.06 (SO MAYBE 2.055)

(ANY OF THESE ARE REASONABLE)

TO FIND x_0 SUCH THAT $P(x \leq x_0) = .98$ WE SET $z_0 = \frac{x_0 - \mu}{\sigma}$

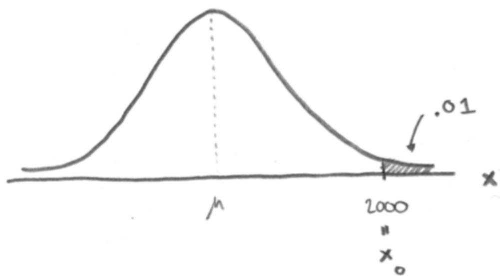
$\Rightarrow 2.05 = \frac{x_0 - 100}{15} \Rightarrow x_0 - 100 = 30.75 \Rightarrow \boxed{x_0 = 130.75}$

(OR 130.9 OR 130.825)

(IF WE ASSUME SCORES ARE DISCRETE THEN A PERSON MUST SCORE 131.)

3. (8 points) A grain loader can be set to discharge grain in amounts that are normally distributed, with mean μ bushels and standard deviation $\sigma = 25.7$ bushels. If a company wishes to use the loader to fill containers that hold 2000 bushels of grain and wants to overfill only one container in 100, at what value of μ should the company set the loader? *(FYI: 1 bushel = 4 pecks = 8 gallons)*

FIND μ :



$.01 = P(x \geq 2000)$

FIRST FIND CORRESPOND z_0 : $P(z \geq z_0) = .01$

$\Rightarrow P(z \leq z_0) = .99$

$\Rightarrow \underline{\underline{z_0 \approx 2.33}}$

THEN $z_0 = \dfrac{x_0 - \mu}{\sigma}$

$2.33 = \dfrac{2000 - \mu}{25.7}$

$59.881 = 2000 - \mu$

$\boxed{\mu = 1940.119 \text{ BUSHELS}}$

4. Consider a (discrete) binomial random variable x with $n = 15$ and $p = .4$.

(a) (2 points) What are the mean μ and the standard deviation σ for x?

$\mu = np = (15)(.4) = \boxed{6}$ $\sigma = \sqrt{npq} = \sqrt{(15)(.4)(.6)} = \sqrt{3.6} \approx \boxed{1.90}$

(b) (2 points) What "rule of thumb" lets us know that the distribution of x can be approximated by a (continuous) normal distribution?

$\boxed{np \geq 5 \text{ AND } nq \geq 5}$ CHECKS OUT: $(15)(.4) = 6$ ✓
$(15)(.6) = 9$ ✓

(c) (8 points) Use the normal approximation to find the following probabilities.

i. $P(x < 5)$

$\approx P_{NORM}(x \leq 4.5) = P(z \leq -.79) = \boxed{.2148}$

ii. $P(x \leq 5)$

$\approx P_{NORM}(x \leq 5.5) = P(z \leq -.26) = \boxed{.3974}$

iii. $P(x > 5)$

$\approx P_{NORM}(x \geq 5.5) = 1 - P_{NORM}(x \leq 5.5)$

$= 1 - .3974 = \boxed{.6026}$

iv. $P(x \geq 5)$

$\approx P_{NORM}(x \geq 4.5) = 1 - P(x \leq 4.5)$

$= 1 - .2148 = \boxed{.7852}$

5. (8 points) Data collected over a long period of time show that a particular genetic defect occurs in 1 of every 1000 children. The records of a medical clinic show $x = 70$ children with the defect in a total of 50,000 examined. If the 50,000 children were a random sample from the population of children represented by past records, what is the probability of observing a value of x equal to 70 or more? Would you say that the observation of $x = 70$ children with genetic defects represents a rare event?

$n = 50{,}000$

$p = .001$ $\Rightarrow$ $\mu = np = 50$

$q = .999$ $\quad \sigma = \sqrt{npq} \approx 7.0675$

$$P(x \geq 70) = P(z \geq 2.83)$$
$$= 1 - P(z \leq 2.83)$$
$$= 1 - .9977 = \boxed{.0023}$$

YES, I WOULD SAY THIS REPRESENTS A RARE EVEN (0.23% LIKELYHOOD)

SINCE n IS SO LARGE, EITHER WAY IS OK.

SINCE x IS BINOMIAL, A MORE ACCURATE APPROXIMATION IS

$$P(x \geq 70) \approx P_{NORM}(x \geq 69.5) = P(z \geq 2.60)$$
$$= 1 - P(z \leq 2.60)$$
$$= 1 - .9953 = \boxed{.0047}$$

STILL UNLIKELY