Name: *Answer Key* **Due 11/27/2019**

Math 173 Introduction to Probability and Statistics **Midterm Exam**

Answer all of the following questions and show enough work that it is clear how you arrived at your answers. Please put a box around your final answers. You may discuss the problems with other students, but your written solutions should be your own. Calculators are allowed, but not required. A standard normal distribution table is provided at the end of the exam. Answers may be left as fractions and/or expressions that may contain square-root ($\sqrt{\cdot}$), factorial (!), permutation (P_r^n), and combination (C_r^n) notation, unless otherwise indicated. Good luck!

1. You are given a *sample* of $n = 6$ measurements:

$$3.1 \quad 3.1 \quad 3.5 \quad 3.6 \quad 3.8 \quad 3.9$$

(a) (2 points) What is the median m?

$$m = \frac{3.5 + 3.6}{2} = \boxed{3.55}$$

(b) (4 points) What is the mean $\bar{x}$?

$$\bar{x} = \frac{\Sigma x}{n} = \frac{21}{6} = \boxed{3.5}$$

(c) (2 points) What is the mode M?

$$M = \boxed{3.1}$$

(d) (4 points) What is the variance s^2?

x	$x - \bar{x}$	$x - \bar{x}^2$
3.1	-.4	.16
3.1	-.4	.16
3.5	0	0
3.6	.1	.01
3.8	.3	.09
3.9	.4	+ .16
		.58

$$s^2 = \frac{\Sigma(x - \bar{x})^2}{n - 1} = \frac{.58}{6 - 1}$$

$$= \boxed{.116}$$

(e) (4 points) What is the standard deviation s?

$$s = \sqrt{s^2} = \sqrt{.116} \approx \boxed{.3406}$$

2. An experiment can result in none, one, or both of the events A and B with the probabilities shown in the following table.

	A	A^c
B	.24	.16
B^c	.36	.24

(a) (4 points) Find $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= .6 + .4 - .24 = \boxed{.76}$$

(b) (4 points) Find $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.24}{.4} = \boxed{.6}$$

(c) (4 points) Are A and B independent? Why?

$\boxed{\text{YES. } P(A|B) = P(A).}$

ALT: $P(A \cap B) = P(A)P(B)$

(d) (4 points) Are A and B mutually exclusive? Why?

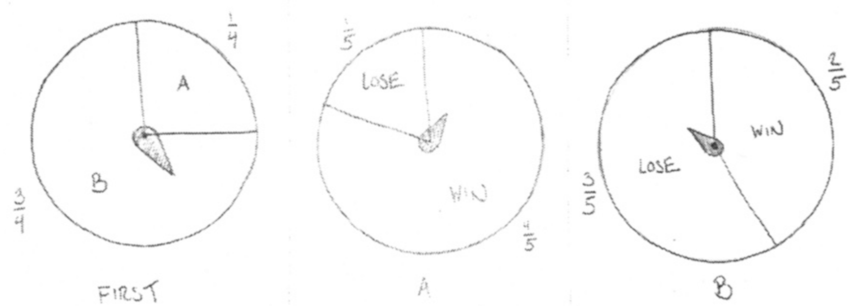
$\boxed{\text{No. } P(A \cap B) \neq 0}$.

3. (4 points) Your band has just recorded 11 songs and is now going to release 8 of them as an album. How many possible ways are there to choose and order 8 (out of the 11 recorded) songs for the album? For this question, your answer can be left in terms of factorials (!) but no combination or permution notation (i.e. C_r^n or P_r^n) is allowed.

$$P^{11}_{8} = \frac{11!}{3!\,8!} = \boxed{6{,}652{,}800}$$

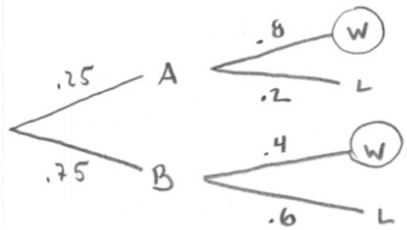
4. Suppose you play a game in which you spin two out of three spinners. The game is played as follows.

- First you spin the spinner labeled FIRST, which will land on A with probability .25 and on B with probability .75. This is your first spin.
- If the FIRST spinner lands on A, then you spin the spinner labeled A, which will land on WIN with probability .8 and on LOSE with probability .2. This is your second spin, and the game is now over.
- If the FIRST spinner lands on B, then you spin the spinner labeled B, which will land on WIN with probability .4 and on LOSE with probability .6. This is your second spin, and the game is now over.
- If your second spin landed on WIN then you win. If your second spin landed on LOSE then you lose.



(a) (6 points) What is the probability that you win this game?

$$P(W) = P(A)P(W|A) + P(B)P(W|B) = (.25)(.8) + (.75)(.4) = \boxed{.5}$$



(LAW OF TOTAL PROB.)

(b) (6 points) What is the probability that your first spin lands on A, given that you win the game?

$$P(A|W) = \frac{P(A)P(W|A)}{P(W)} = \frac{(.25)(.8)}{.5} = \boxed{.4}$$

(BAYES' RULE)

5. (8 points) Medical case histories indicate that different illnesses may produce identical symptoms. Suppose a particular set of symptoms, which we will denote as event H, occurs only when any one of three illnesses – A, B or C – occurs. (For simplicity, we will assum that illnesses A, B, and C are mutually exclusive.) Studies show these probabilities of getting the three illnesses:

$$P(A) = .01, \quad P(B) = .005, \quad P(C) = .02.$$

The probabiities of developing the symptoms H, given a specific illness are

$$P(H|A) = .90, \quad P(H|B) = .95, \quad P(H|C).75.$$

Assuming that an ill person shows the symptoms H, what is the probability that the person has illness A?

$$P(A|H) = \frac{P(A)P(H|A)}{P(H)} = \frac{P(A)P(H|A)}{P(A)P(H|A) + P(B)P(H|B) + P(C)P(H|C)}$$

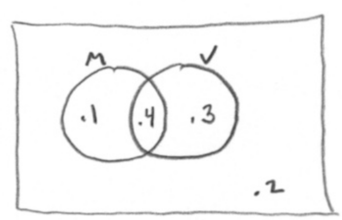
$$= \frac{(.01)(.90)}{(.01)(.90) + (.005)(.95) + (.02)(.75)}$$

$$= \frac{.009}{.02875} \approx \boxed{.3130}$$

(BAYES RULE + LAW OF TOTAL PROB)

6. (4 points) A particular pizzeria sells plain pizzas and pizzas with meat toppings, vegetable toppings, or both meat and vegetable toppings. Suppose 50% of pizzas sold have meat toppings, 70% have vegetable toppings, and 20% have no toppings (plain). What percentage of pizzas sold have both meat and vegetable toppings?

$$P(M \cup V) = 1 - P(M^c \cap V^c) = 1 - .2 = .8$$



$$\therefore \quad P(M \cup V) = P(M) + P(V) - P(M \cap V)$$

$$.8 = .5 + .7 - P(M \cap V)$$

$$P(M \cap V) = .5 + .7 - .8 = .4 \rightarrow \boxed{40\%}$$

7. (8 points) An insurance company offers factory workers a one-year life insurance policy for d dollars. If a worker with insurance has a fatal accident on the job, then the insurance company pays her family \$150,000. Suppose each year 1% of workers at this factory have a fatal accident on the job. If the insurance company's expected gain (profit) per policy sold is to be \$1,000, what price should they charge for a one-year policy?

LET x = PROFIT TO INSURANCE COMPANY (RANDOM VARIABLE)

x	$p(x)$
d	.99
$d - 150{,}000$	.01

$$E[x] = \underbrace{\sum x\, p(x)}_{} = 1{,}000$$

$$d(.99) + (d - 150{,}000)(.01) = 1{,}000$$

$$d - 1500 = 1{,}000$$

$$d = \boxed{\$2{,}500}$$

8. (5 points) The life span of fruit flies is a continuous, normally distributed random variable with mean $\mu = 40$ days and standard deviation $\sigma = 7$ days. Find the probability that a fruit fly lives between 38 and 43 days.

$$P(38 \le x \le 43) = P(x \le 43) - P(x \le 38)$$

$$= P\left(z \le \frac{43-40}{7}\right) - P\left(z \le \frac{38-40}{7}\right)$$

$$\approx P(z \le .43) - P(z \le -.29)$$

$$\approx .6664 - .3859 = \boxed{.2805}$$

(CALC ANSWER : .2783)

9. Suppose every time a person goes fishing, they have a 65% chance of catching a fish.

(a) (4 points) If this person goes fishing 7 days in a row, what is the probability that this person catches a fish on exactly 4 of these days?

BINOMIAL: $n = 7$

$p = .65$

$q = .35$

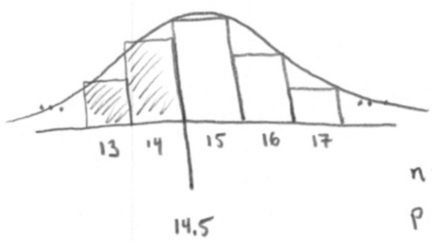
x = # SUCCESSES

$$P(x) = C^n_x \, p^x \, q^{n-x}$$

$$P(4) = C^7_4 \, (.65)^4 (.35)^3$$

$$= \boxed{.2679}$$

(b) (6 points) If this person goes fishing 30 days in a row, use a normal distribution to approximate the probability that this person catches a fish on fewer than 15 of these days.



$n = 30$

$p = .65$

$q = .35$

$$\mu = np = (30)(.65) = 19.5$$

$$\sigma = \sqrt{npq} = \sqrt{(30)(.65)(.35)} = 2.6125$$

$$P(x < 15) \approx P(x_{NORM} \le 14.5) = P\left(z \le \frac{14.5 - 19.5}{2.6125}\right)$$

$$\approx P(z \le -1.91) \approx \boxed{.0281}$$

(c) (2 points) What "rule of thumb" allows us to use a normal approximation in part (b)?

$$\boxed{\begin{array}{l} np \ge 5 \\ nq \ge 5 \end{array}}$$

$(30)(.65) = 19.5 \ge 5$ ✓

$(30)(.35) = 10.5 \ge 5$ ✓

10. (5 points) Two animals are to be randomly selected from a group of 4 goats and 8 sheep. Let x equal the number of goats selected. Describe the probability distribution for the random variable x by filling in the following chart with all possible values x and the corresponding probabilities $p(x)$.

x	0	1	2
$p(x)$	$\frac{C^4_0 C^8_2}{C^{12}_2} = .4242$	$\frac{C^4_1 C^8_1}{C^{12}_2} = .4848$	$\frac{C^4_2 C^8_0}{C^{12}_2} = .0909$

11. (5 points) A random sample of 40 inner tubes produced by a particular manufacturer are tested to see at what pressure they fail. The sample has mean $\bar{x} = 165$ lbs/in^2 with a standard deviation of $s = 8$ lbs/in^2. Acording to Chebyshev's Theorem, at least how many of the 40 inner tubes tested failed at a pressure between 149 lbs/in^2 and 181 lbs/in^2 ($149 \leq x \leq 181$)?

$165 - 16$ $\quad$ $165 + 16$ $\quad$ WITHIN $k = 2$ STAND. DEV. OF MEAN

$\bar{x} - 2s$ $\quad$ $\bar{x} + 2s$

$$1 - \frac{1}{k^2} = 1 - \frac{1}{2^2} = .75 \longrightarrow 75\% \text{ OF } 40 \longrightarrow \boxed{\geq 30}$$

12. (5 points) Suppose you have a music collection of 10,000 songs with an average length of 251 seconds and a stanfard deviation of 25 seconds. You randomly select 50 songs from your music library. Use the central limit theorem to approximate the probability that the 50 songs have a total length of more than 3 hours 23 minutes and 20 seconds (i.e. 12,200 seconds).

SAMPLE DISTRIBUTION FOR Σx HAS MEAN $n\mu = (50)(251) = 12550$

STANDARD ERROR $\sigma\sqrt{n} = (25)\sqrt{50} \approx 176.7767$

$$P(\Sigma x \geq 12200) = P\left(z \geq \frac{12200 - 12550}{176.7767}\right) \approx 1 - P(z \leq -3.11)$$

$$\approx 1 - .0009 = \boxed{.9991}$$