Name: * Answer Key *

**Due 11/27/2019**

**Math 173 Introduction to Probability and Statistics**

**Midterm Exam**

Answer all of the following questions and show enough work that it is clear how you arrived at your answers. Please put a box around your final answers. You may discuss the problems with other students, but your written solutions should be your own. Calculators are allowed, but not required. A standard normal distribution table is provided at the end of the exam. Answers may be left as fractions and/or expressions that may contain square-root ($\sqrt{\cdot}$), factorial (!), permutation ($P_r^n$), and combination ($C_r^n$) notation, unless otherwise indicated. Good luck!

1. You are given a *sample* of $n = 6$ measurements:

   3.1 3.1 3.5 3.6 3.8 3.9

   (a) (2 points) What is the median $m$?

   $$m = \frac{3.5 + 3.6}{2} = \boxed{3.55}$$

   (b) (4 points) What is the mean $\bar{x}$?

   $$\bar{x} = \frac{\Sigma x}{n} = \frac{21}{6} = \boxed{3.5}$$

   (c) (2 points) What is the mode $M$?

   $$M = \boxed{3.1}$$

   (d) (4 points) What is the variance $s^2$?

| $x$ | $x - \bar{x}$ | $x - \bar{x}^2$ |
|---|---|---|
| 3.1 | -.4 | .16 |
| 3.1 | -.4 | .16 |
| 3.5 | 0 | 0 |
| 3.6 | .1 | .01 |
| 3.8 | .3 | .09 |
| 3.9 | .4 | + .16 |
| | | .58 |

   $$s^2 = \frac{\Sigma (x - \bar{x})^2}{n - 1} = \frac{.58}{6 - 1}$$

   $$= \boxed{.116}$$

   (e) (4 points) What is the standard deviation $s$?

   $$s = \sqrt{s^2} = \sqrt{.116} \approx \boxed{.3406}$$

2. An experiment can result in none, one, or both of the events $A$ and $B$ with the probabilities shown in the following table.

| | $A$ | $A^c$ |
|---|---|---|
| $B$ | .24 | .16 |
| $B^c$ | .36 | .24 |

(a) (4 points) Find $P(A \cup B)$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= .6 + .4 - .24 = \boxed{.76}$

(b) (4 points) Find $P(A|B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.24}{.4} = \boxed{.6}$

(c) (4 points) Are $A$ and $B$ independent? Why?

YES. $P(A|B) = P(A)$.

ALT: $P(A \cap B) = P(A)P(B)$

(d) (4 points) Are $A$ and $B$ mutually exclusive? Why?

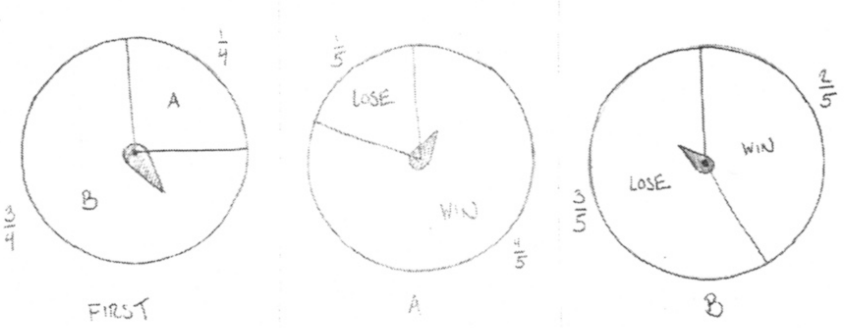
No. $P(A \cap B) \neq 0$

3. (4 points) Your band has just recorded 11 songs and is now going to release 8 of them as an album. How many possible ways are there to choose and order 8 (out of the 11 recorded) songs for the album? For this question, your answer can be left in terms of factorials (!) but no combination or permution notation (i.e. $C_r^n$ or $P_r^n$) is allowed.

$P_8^{11} = \frac{11!}{3!\,8!} = \boxed{6,652,800}$

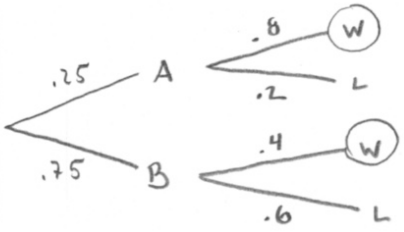
4. Suppose you play a game in which you spin two out of three spinners. The game is played as follows.

- First you spin the spinner labeled FIRST, which will land on A with probability .25 and on B with probability .75. This is your first spin.
- If the FIRST spinner lands on A, then you spin the spinner labeled A, which will land on WIN with probability .8 and on LOSE with probability .2. This is your second spin, and the game is now over.
- If the FIRST spinner lands on B, then you spin the spinner labeled B, which will land on WIN with probability .4 and on LOSE with probability .6. This is your second spin, and the game is now over.
- If your second spin landed on WIN then you win. If your second spin landed on LOSE then you lose.




(a) (6 points) What is the probability that you win this game?

$$P(W) = P(A)P(W|A) + P(B)P(W|B) = (.25)(.8) + (.75)(.4) = \boxed{.5}$$




(LAW OF TOTAL PROB.)

(b) (6 points) What is the probability that your first spin lands on A, given that you win the game?

$$P(A|W) = \frac{P(A)P(W|A)}{P(W)} = \frac{(.25)(.8)}{.5} = \boxed{.4}$$

(BAYES' RULE)