Name: * Answer Key * 10/2/2019

Math 173-CD Quiz 2

Please put away all papers and electronic devices except for a calculator. Show enough work that it is clear how you arrived at your answer. Answers can be given as frations or decimals rounded to 4 decimal places. Box/circle your final answers. Good luck!

1. An experimaent can result in 6 possible simple events

$$S = \{E_1, E_2, E_3, E_4, E_5, E_6\}$$

with the following probabilities.

$$P(E_1) = .05, \quad P(E_2) = .10, \quad P(E_3) = .25,$$

$$P(E_4) = P(E_5) = P(E_6).$$

Suppose the events A and B are defined as follows.

$$A = \{E_1, E_3, E_4\}$$
$$B = \{E_1, E_2, E_3\}$$

(a) (4 points) Find the probability $P(E_4)$.

Since $P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) = 1$

$\Rightarrow \quad .05 + .10 + .25 + 3P(E_4) = 1$

$3P(E_4) = 1 - .4 = .6$

$\boxed{P(E_4) = .2}$

(b) (4 points) Find the probability $P(B)$

$P(B) = P(\{E_1, E_2, E_3\}) = P(E_1) + P(E_2) + P(E_3)$

$= .05 + .10 + .25 = \boxed{.4}$

(c) (4 points) Find the probability $P(A \cap B)$.

$A \cap B = \{E_1, E_3\} \Rightarrow P(A \cap B) = P(E_1) + P(E_3)$

$= .05 + .25 = \boxed{.3}$

(d) (4 points) Find the probability $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.3}{.4} = \boxed{.75}$$

(e) (4 points) Are A and B mutually exclusive? Why or why not?

No, $P(A \cap B) \neq 0$

(f) (4 points) Are A and B independent? Why or why not?

$\boxed{\text{No} \quad P(A) \neq P(A|B)}$

$P(E_1) + P(E_3) + P(E_4) \neq P(A|B)$

$.5 = .05 + .25 + .2 \neq .75$

OR

$P(A \cap B) \neq P(A)P(B)$

$.3 \neq (.5)(.4)$

$.3 \neq .2$

2. Provide formulas for the following.

   (a) (4 points) The number of ways to choose and arrange r distinct objects from a collection of n distinct objects, i.e. P_r^n.

   $$\frac{n!}{(n-r)!}$$

   (b) (4 points) The number of ways to choose r distinct objects from a collection of n distinct objects, i.e. C_r^n.

   $$\frac{n!}{r!(n-r)!}$$

3. (4 points) How many ways can the letters Q, W, E, R, T, and Y be ordered?

$$P_6^6 = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720}$$

4. (4 points) How many ways are there to select 3 (different) side dishes from a restaurant that serves 6 side dishes?

$$C_3^6 = \frac{6!}{3!\,3!} = \frac{720}{(6)(6)} = \boxed{20}$$

5. Suppose a particular club has 20 members.

   (a) (6 points) How many ways can the club select 4 people to work on a particular project?

$$C_4^{20} = \frac{20!}{4!\,16!} = \boxed{4845}$$

   (b) (6 points) How many ways can the club elect a president, vice president, and secretary?

$$P_3^{20} = \frac{20!}{17!} = 20 \cdot 19 \cdot 18 = \boxed{6840}$$

6. (8 points) How many ways are there for Noah to select 2 sheep, 2 goats, and 2 camels from a group of 5 sheep, 6 goats, and 7 camels?

CHOOSE SHEEP CHOOSE GOATS CHOOSE CAMELS

$$C_2^5 \cdot C_2^6 \cdot C_2^7$$

$$(10)(15)(21) = \boxed{3150}$$

7. A drawer contains 7 new batteries and 3 dead batteries. You grab 2 of the batteries.

(a) (8 points) What is the probability that they are both dead?

WAYS TO SELECT 2 DEAD BATTERIES: $C_2^3 = 3$

WAYS TO SELECT 2 BATTERIES: $C_2^{10} = 45$

$$P(\text{2 DEAD}) = \frac{3}{45} = \boxed{\frac{1}{15}} \approx .0667$$

$P(1^{ST} \text{ DEAD} \cap 2^{nd} \text{ DEAD}) =$

OR $P(1^{ST} \text{ DEAD}) P(2^{nd} \text{ DEAD} \mid 1^{ST} \text{ DEAD})$

$= \left(\frac{3}{10}\right)\left(\frac{2}{9}\right)$

$= \frac{6}{90} = \frac{1}{15}$ ✓

(b) (8 points) What is the probability that at least one of the batteries is new?

THIS IS THE OPPOSITE OF THE EVENT IN PART (a).

$\therefore$ $P(\text{AT LEAST ONE NEW BATT.}) = 1 - P(\text{ALL DEAD BATT.})$

$$= 1 - \frac{1}{15}$$

$$= \boxed{\frac{14}{15}} \approx .9333$$

8. An experiment can result in events A, B, both A and B, or neither with the following probabilities.

	A	A^c
B	.16	.56
B^c	.06	.22

(a) (4 points) Find $P(A)$.

$P(A) = P(A \cap B) + P(A \cap B^c) = .16 + .06 = \boxed{.22}$

(b) (6 points) Find $P(A|B)$.

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.16}{.16 + .56} = \dfrac{.16}{.72} = \boxed{.2222}$

(c) (6 points) Are A and B independent? Why or why not?

No, $P(A) \neq P(A|B)$

$.22 \neq .2222$

ALSO $P(A \cap B) \neq P(A)\,P(B)$

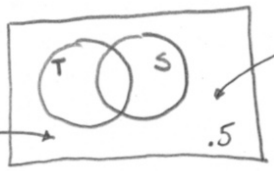
$(.16) \neq (.22)(.72) = .1584$

9. (8 points) Suppose the probability that a pea plant is tall is .4, the probability that a pea plant has smooth leaves is .25, and the probabilty that a pea plant is neither tall nor has smooth leaves is .5. Find the probability that a pea plant is both tall and has smooth leaves.

$P(T) = .4$

$P(S) = .25$

$P(T^c \cap S^c) = .5$



$P((T \cup S)^c) = .5$

$\therefore P(T \cup S) = .5$

ADDITION RULE: $P(T \cup S) = P(T) + P(S) - P(T \cap S)$

$.5 = .4 + .25 - P(T \cap S)$

$P(T \cap S) = .4 + .25 - .5 = \boxed{.15}$