Name: * Answer Key * **11/6/2019**
Math 173-FG **Quiz 3**

Please put away all papers and electronic devices except a calculator. Show enough work that it is clear how you arrived at your answer. Correct answers with no work shown will not receive full credit. Box/circle your final answers. Good luck!

1. (10 points) Suppose 20% of apartments that are owned have a doorman, and 4% of apartments that are rented have a doorman. If 25% of apartments are owned and 75% are rented, what percentage of apartmets (overall) have a doorman?

Let S_1 = APARTMENT IS OWNED
S_2 = APARTMENT IS RENTED
A = APARTMENT HAS DOORMAN

GIVEN: $P(A|S_1) = .2$ $P(S_1) = .25$
$P(A|S_2) = .04$ $P(S_2) = .75$

FIND: $P(A)$

LAW OF TOTAL PROBABILITY: $P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2)$

$= (.25)(.2) + (.75)(.04)$

$= .05 + .03 = \boxed{.08 \text{ or } 8\%}$

2. Suppose that on a particular day, 15% of CCNY students drove to school, 75% took the subway, and 10% walked or rode a bike. Furthermore, 10% of those who drove to school were late, 20% of those who toook the subway were late, and 5% of those who walked or biked were late.

(a) (8 points) Find the probability that a CCNY student was late on this day.

Let S_1 = DROVE
S_2 = SUBWAY
S_3 = WALK/BIKE
A = LATE

GIVEN: $P(S_1) = .15$ $P(A|S_1) = .1$
$P(S_2) = .75$ $P(A|S_2) = .2$
$P(S_3) = .1$ $P(A|S_3) = .05$

FIND: $P(A)$

LAW OF TOTAL PROBABILITY: $P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + P(S_3)P(A|S_3)$

$= (.15)(.1) + (.75)(.2) + (.1)(.05) = \boxed{.17}$

$= .015 + .15 + .005$

(b) (10 points) If a student was late, what is the probability that they took the subway to school?

BAYES' RULE: $P(S_2|A) = \dfrac{P(S_2)P(A|S_2)}{P(A)}$ ← CALCULATED IN PART (a)

$= \dfrac{(.75)(.2)}{.17} = \boxed{.8824}$

3. Every time you play a Youtube video, a video ad plays. Suppose the ad is chosen randomly and 30% of ads are 5 seconds long, 45% are 15 seconds long, and 25% are 30 seconds long. Let x equal the length of the randomly selected ad in seconds.

(a) (8 points) Describe the probability distribution $p(x)$ by filling in the chart below.

x	5	15	30
$p(x)$	.3	.45	.25

(b) (8 points) Calculate the expected value $E(x)$ for x.

$$\mu = E[x] = \sum x\, p(x)$$

$$= (5)(.3) + (15)(.45) + (30)(.25)$$

$$= 1.5 + 6.75 + 7.5$$

$$= \boxed{15.75}$$

4. A raffle is being held in which 600 tickets are sold for \$5 each. There is 1 first prize of \$1500 and there are 2 second prizes of \$500 each. All other tickets receive no prize (\$0). Let x equal the net gain/loss from buying one ticket, that is

$$x = \text{prize money} - 5.$$

(a) (8 points) Describe the probability distribution $p(x)$ by filling in the chart below.

x	1495	495	-5
$p(x)$	$\frac{1}{600}$	$\frac{2}{600}$	$\frac{597}{600}$

(b) (8 points) Calculate the expected value $E(x)$ for x.

$$\mu = E[x] = \sum x\, p(x)$$

$$= (1495)\left(\frac{1}{600}\right) + (495)\left(\frac{2}{600}\right) + (-5)\left(\frac{597}{600}\right)$$

$$= \boxed{-.8333}$$

"ON AVERAGE, EVERYONE WHO BUYS ONE TICKET LOSES \$0.83"

5. (10 points) If a fair coin is flipped 9 times, find the probability that exactly 3 heads are observed.

BINOMIAL EXPERIMENT : $n = 9$ $\quad P(x=k) = C^n_k \, p^k \, q^{n-k}$

$p = .5$

$q = .5$

x = # HEADS IN 9 TRIALS

$P(x=3) = \underbrace{C^9_3}_{(84)} (.5)^3 (.5)^6 = \boxed{.1641}$

6. In a certain laboratory, every bacterial culture has a 15% chance of becoming contaminated, independent of the other cultures. Suppose this lab grows 30 bacterial cultures. Let x equal the number of contaminated cultures.

(a) (8 points) Find the probability that exactly $x = 4$ of the cultures become contaminated.

BINOMIAL EXPERIMENT : $n = 30$ $\quad P(x=k) = C^n_k \, p^k \, q^{n-k}$

$p = .15$

$q = .85$

x : # CONTAMINATED CULTURES OUT OF 30

$P(x=4) = \underbrace{C^{30}_4}_{(27405)} (.15)^4 (.85)^{26} = \boxed{.2028}$

(b) (5 points) Find the expected value $E[x] = \mu$ for the number of contaminated cultures x.

$\mu = E[x] = np = (30)(.15) = \boxed{4.5}$

(c) (5 points) Find the standard deviation σ for the number of contaminated cultures x.

$\sigma = \sqrt{npq} = \sqrt{(30)(.15)(.85)} = \sqrt{3.825} = \boxed{1.9558}$

7. (12 points) Suppose a shipment of 12 computer monitors contains 7 standard monitors and 5 high-definition monitors. Three computer monitors are selected at random. Let x be the number of high-definition montors selected. Describe the probability distribution $p(x)$ by filling in the chart below.

x	0	1	2	3
$p(x)$	$\frac{C^5_0 C^7_3}{C^{12}_3}$	$\frac{C^5_1 C^7_2}{C^{12}_3}$	$\frac{C^5_2 C^7_1}{C^{12}_3}$	$\frac{C^5_3 C^7_0}{C^{12}_3}$
	.1591	.4773	.3182	.0455