Name: *Answer Key*   12/3/2018

Math 173 Introduction to Probability and Statistics   **Quiz 4**

1. Suppose a random sample of $n = 25$ observations is selected form a population that is normally distributed with mean $\mu = 106$ and standard deviation $\sigma = 12$.

   (a) (8 points) Give the mean and standard deviation (i.e. standard error) of the sampling distribution of the sample mean $\bar{x}$.

   MEAN $\boxed{106}$

   $$\text{S.E.} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{25}} = \boxed{2.4}$$

   (b) (8 points) Find the probability that $\bar{x}$ exceeds 110.

   $$z = \frac{\bar{x} - \mu}{\text{S.E.}} = \frac{110 - 106}{2.4} \approx 1.67$$

   $$P(\bar{x} \geq 110) = 1 - P(z \leq 1.67) = 1 - .9525 = \boxed{.0475}$$

2. A random sample of size $n = 50$ is selected from a binomial distribution with population proportion $p = .7$.

   (a) (8 points) What will be the mean and standard deviation (i.e. standard error) of the sampling distribution of $\hat{p}$?

   MEAN $\boxed{.7}$

   $$\text{S.E.} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.7)(.3)}{50}} \approx \boxed{.0648}$$

   (b) (8 points) Find the probability that the sample proportion $\hat{p}$ is less than .8.

   $$z = \frac{\hat{p} - p}{\text{S.E.}} = \frac{.8 - .7}{.0648} \approx 1.54$$

   $$P(\hat{p} \leq .8) = P(z \leq 1.54) = \boxed{.9382}$$

3. The meat department of a local supermarket chain packages ground beef using meat trays designed to hold approximately 1 pound of meat. A random sample of 35 packages produced weight measurements with an average of 1.01 pounds and a standard deviation of .18 pound.

(a) (8 points) Construct a 90% confidence interval for the average weight of all packages sold in the smaller meat trays by this supermarket chain.

$$\mu \approx \bar{x} \pm z_{\alpha/2}\, S.E. \quad , \quad \alpha = .1 \Rightarrow P(z \le z_{\alpha/2}) = .95$$

$$\Rightarrow z_{\alpha/2} = 1.645$$

$$\mu \approx 1.01 \pm \underbrace{1.645 \left(\overbrace{\frac{.18}{\sqrt{35}}}^{.0304} \right)}_{.0501} \approx \boxed{[.9599 \ , \ 1.0601]}$$

(b) (4 points) What does the phrase "90% confident" mean?

IF WE WERE TO REPEAT IS EXPERIMENT OVER & OVER WITH DIFFERENT RANDOM SAMPLES OF PACKAGES, WE EXPECT 90% OF THE GENERATED INTERVALS TO CONTAIN THE TRUE POPULATION MEAN.

4. A sample survey is designed to estimate the proportion of sports utility vehicles being driven in the state of California. A random sample of 500 registrations are selected from a Department of Motor Vehicles database, and 68 are classified as sports utility vehicles.

(a) (8 points) Use a 95% confidence interval to estimate the proportion of sports utility vehicles in California.

$$p \approx \hat{p} \pm z_{\alpha/2}\, S.E. \quad , \quad \hat{p} = \frac{x}{n} = \frac{68}{500} = .136$$

$$\alpha = .05 \quad , \quad p(z \ge z_{\alpha/2}) = .025 \Rightarrow z_{\alpha/2} = 1.96 \quad , \quad S.E \approx \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$p \approx .136 \pm \underbrace{1.96\sqrt{\frac{(.136)(.864)}{500}}}_{.0300} \approx \boxed{[.1060 \ , \ .1660]}$$

(b) (4 points) How can you estimate the proportion of sports utility vehicles in California with a higher degree of accuracy (i.e. smaller interval)? (HINT: There are two answers.)

(1) LOWER THE CONFIDENCE COEFFICIENT

(2) INCREASE THE SAMPLE SIZE.

5. To compare the effect of stress in the form of noise on the ability to perform a simple task, 70 subjects were divided into two groups. The first group of 30 subjects acted as a control, while the second group of 40 were the experimental group. Although each subject performed the task, the experimental group subjects had to perform the task while loud rock music was played. The time to finish the task was recorded for each subject and the following summary was obtained:

	Control	Experimental
n	30	40
$\bar{x}$	15 min	23 min
s	4 min	10 min

(a) (8 points) Find a 99% confidence interval for the difference in mean completion times for these tow groups.

$$\mu_1 - \mu_2 \approx 15 - 23 \pm 2.58 \underbrace{\sqrt{\frac{4^2}{30} + \frac{10^2}{40}}}_{1.7416} \qquad \left(\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \underbrace{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}_{S.E.}\right)$$

$$\approx -8 \pm 4.4934$$

$$\approx \boxed{[-12.4934, -3.5066]}$$

(b) (4 points) Based on the confidence interval is part (a), is there sufficient evidence to indicate a difference in the average time to completion for the two groups? Explain briefly.

$\boxed{\text{YES}}$, SINCE THE CONFIDENCE INTERVAL DOES NOT CONTAIN 0.

$\mu_1 - \mu_2 < 0$ (NEGATIVE)

$\Rightarrow \mu_1 < \mu_2$.

6. In a study to compare the effects of two pain relievers it was found that of the $n_1 = 200$ randomly selected individuals who used the first pain reliever, 93% indicated that it relieved their pain. Of the $n_2 = 450$ randomly selected individuals who used the second pain reliever, 96% indicated that it relieved their pain.

(a) (8 points) Find a 99% confidence interval for the difference in the proportions experiencing relief from pain for these two pain relievers.

$$p_1 - p_2 \approx \hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2} \underbrace{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}}_{S.E.}$$

$$\approx .93 - .96 \pm 2.58 \underbrace{\sqrt{\frac{(.93)(.07)}{200} + \frac{(.96)(.04)}{450}}}_{.0203}$$

$$\approx -.03 \pm .0523 \approx \boxed{[-.0823, .0223]}$$

(b) (4 points) Based on the confidence interval in part (a), is there sufficient evidence to indicate a difference in the proportions experiencing relief for the two pain relievers? Explain briefly.

$\boxed{\text{No}}$, THE CONFIDENCE INTERVAL CONTAINS 0.

SO IT MAY BE THAT $\mu_1 - \mu_2 = 0$

$\Rightarrow \mu_1 = \mu_2$.