

* ANSWER KEY *

Directions Answer all questions in the space provided. Show all work and box your final answers. Answers with no work shown will not receive full credit. Good luck!

1. Compute and simplify the following.

(a) (4 points) $\left(3 + \frac{1}{4}\right)\left(1 - \frac{4}{5}\right)$

$$= \left(\frac{12}{4} + \frac{1}{4}\right)\left(\frac{5}{5} - \frac{4}{5}\right) = \frac{13}{4} \cdot \frac{1}{5} = \boxed{\frac{13}{20}}$$

(b) (4 points) $\frac{3}{4} - \frac{3}{4/5}$

$$= \frac{3}{4} - \frac{3}{4/5} = \frac{3}{4} \cdot \frac{5}{5} - \frac{5}{5} \cdot \frac{3}{4/5}$$

$$= \frac{3}{20} - \frac{15}{4} = \frac{3}{20} - \frac{75}{20} = -\frac{72}{20} = \boxed{-\frac{18}{5}}$$

(c) (4 points) $\frac{\frac{3}{7} + \frac{1}{3}}{\frac{1}{21} + \frac{2}{14}}$ LCD = 42

$$= \frac{42\left(\frac{3}{7} + \frac{1}{3}\right)}{42\left(\frac{1}{21} + \frac{2}{14}\right)} = \frac{18 + 14}{2 + 6} = \frac{32}{8} = \boxed{4}$$

2. Express the domain of the following expressions using interval notation.

(a) (4 points) $\sqrt{x+7}$

$$x+7 \geq 0$$

$$x \geq -7$$

$$\boxed{[-7, \infty)}$$

(b) (4 points) $\frac{x}{5-x}$

$$5-x \neq 0$$

$$5 \neq x$$

$$\boxed{(-\infty, 5) \cup (5, \infty)}$$

3. Simplify the following expressions and eliminate any negative exponents.

(a) (4 points) $\sqrt{27} + \sqrt{75}$

$$= \sqrt{9} \sqrt{3} + \sqrt{25} \sqrt{3}$$

$$= 3\sqrt{3} + 5\sqrt{3} = \boxed{8\sqrt{3}}$$

(b) (4 points) $\left(\frac{3x^3y^{-2}}{4x^{-3}y^2}\right)^{-2} = \left(\frac{3x^6}{4y^4}\right)^{-2} = \left(\frac{4y^4}{3x^6}\right)^2$

$$= \frac{4^2 (y^4)^2}{3^2 (x^6)^2} = \boxed{\frac{16y^8}{9x^{12}}}$$

4. Compute the product/quotient/sum/difference and simplify.

$$\begin{aligned}
 \text{(a) (4 points)} \quad x^{3/2} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) &= x^{3/2} \left(x^{1/2} - x^{-1/2} \right) \\
 &= x^{3/2 + 1/2} - x^{3/2 - 1/2} = \boxed{x^2 - x}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) (4 points)} \quad \frac{1}{x^2 - x - 6} - \frac{1}{x + 2} - \frac{2}{x - 3} \quad \text{LCD} &= (x - 3)(x + 2) \\
 &\quad \uparrow \\
 &\quad (x - 3)(x + 2) \\
 &= \frac{1}{(x - 3)(x + 2)} - \frac{1}{x + 2} \cdot \frac{(x - 3)}{(x - 3)} - \frac{2}{x - 3} \cdot \frac{(x + 2)}{(x + 2)} \\
 &= \frac{1 - (x - 3) - 2(x + 2)}{(x - 3)(x + 2)} = \frac{1 - x + 3 - 2x - 4}{(x - 3)(x + 2)} = \boxed{\frac{-3x}{(x - 3)(x + 2)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) (4 points)} \quad \frac{x + 3}{4x^2 - 9} \div \frac{x^2 + 7x + 12}{2x^2 + 7x - 15} \\
 &= \frac{\cancel{(x + 3)}}{(2x + 3)\cancel{(2x - 3)}} \cdot \frac{\cancel{(2x - 3)}(x + 5)}{\cancel{(x + 5)}(x + 4)} \\
 &= \boxed{\frac{x + 5}{(2x + 3)(x + 4)}}
 \end{aligned}$$

5. Solve each of the following equations by factoring.

(a) (4 points) $5x^3 - 40x^2 + 60x = 0$

$$5x(x^2 - 8x + 12) = 0$$

$$5x(x - 6)(x - 2) = 0$$

$$\swarrow$$
$$5x = 0$$

$$x = 0$$

$$\searrow$$
$$x - 6 = 0$$

$$x = 6$$

$$\searrow$$
$$x - 2 = 0$$

$$x = 2$$

$$x = 0, 2, 6$$

(b) (4 points) $3x^2 + 4 = -7x$

$$3x^2 + 7x + 4 = 0$$

$$(3x + 4)(x + 1) = 0$$

$$\swarrow$$
$$3x + 4 = 0$$

$$x = -\frac{4}{3}$$

$$\searrow$$
$$x + 1 = 0$$

$$x = -1$$

$$x = -1, -\frac{4}{3}$$

(c) (4 points) $x^3 - 4x^2 - x + 4 = 0$

$$x^2(x - 4) - (x - 4) = 0$$

$$(x^2 - 1)(x - 4) = 0$$

$$(x + 1)(x - 1)(x - 4) = 0$$

$$\swarrow$$
$$x + 1 = 0$$

$$x = -1$$

$$\searrow$$
$$x - 1 = 0$$

$$x = 1$$

$$\searrow$$
$$x - 4 = 0$$

$$x = 4$$

$$x = -1, 1, 4$$

6. Solve each of the following equations.

(a) (4 points) $\frac{x}{2x+7} - \frac{x+1}{x+3} = 1$

LCD = $(2x+7)(x+3)$

$$\frac{x}{2x+7} (2x+7)(x+3) - \frac{x+1}{x+3} (2x+7)(x+3) = 1 (2x+7)(x+3)$$

$$x(x+3) - (x+1)(2x+7) = (2x+7)(x+3)$$

$$x^2 + 3x - 2x^2 - 9x - 7 = 2x^2 + 13x + 21$$

$$-x^2 - 6x - 7 = 2x^2 + 13x + 21$$

$$0 = 3x^2 + 19x + 28$$

$$0 = (3x+7)(x+4)$$

$$x = -\frac{7}{3}$$

$$x = -4$$

$x = -4, -\frac{7}{3}$

(b) (4 points) $\sqrt{9-x} + 1 = x - 6$

$$\sqrt{9-x} = x - 7$$

$$9 - x = (x - 7)^2 = x^2 - 14x + 49$$

$$0 = x^2 - 13x + 40$$

$$0 = (x - 5)(x - 8)$$

$$x = 5$$

$$x = 8$$

$x = 8$

CHECK: $\sqrt{9-5} + 1 \stackrel{?}{=} 5-6$

No.

⊗

CHECK:

$$\sqrt{9-8} + 1 \stackrel{?}{=} 8-6$$

Yes.

✓