Name: *ANSWER KEY* Due 12/19/2016
Math 190-EF Exam 4

Directions Answer all questions in the space provided. Show all work and box your final answers. Turn in this exam in NAC 0/201 on Monday, 12/19/2016, before we take our final exam at 1pm. (Please arrive 15 minutes early.) Good luck!

1. (a) Give an equation for *the unit circle*, that is, the circle with radius 1 centered at the origin.

$$\boxed{x^2 + y^2 = 1}$$

(b) Show (algebraically) that the point $(-3/4, \sqrt{7}/4)$ is on the unit circle.

PLUG IN: $\left(-\frac{3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right) = 1$

$$\frac{9}{16} + \frac{7}{16} = 1$$

$$\frac{16}{16} = 1 \quad \checkmark$$

(c) Let P be a point on the unit circle that lies in quadrant IV and has x-coordinate $5/7$. What is the y-coordinate of P?

$$\left(\frac{5}{7}\right)^2 + y^2 = 1$$

$$y^2 = 1 - \frac{25}{49}$$

$$y = \pm\sqrt{\frac{24}{49}}$$

IN QIV, $y < 0$

SO $\boxed{y = -\frac{2\sqrt{6}}{7}}$

2. For each of the following values of t, find the reference number and the terminal point associated with t.

(a) $t = 7\pi/6$; Reference number = $\frac{\pi}{6}$, Terminal point = ($-\frac{\sqrt{3}}{2}$, $-\frac{1}{2}$).

(b) $t = -11\pi/3$; Reference number = $\frac{\pi}{3}$, Terminal point = ($\frac{1}{2}$, $\frac{\sqrt{3}}{2}$).

(c) $t = 103\pi/2$; Reference number = $\frac{\pi}{2}$, Terminal point = (0, -1).

3. (a) Convert $8\pi/9$ radians to degrees.

$$\frac{8\pi}{9} \cdot \frac{180^\circ}{\pi} = 8 \cdot 20^\circ = \boxed{160^\circ}$$

(b) Convert -405° to radians.

$$-405^\circ \cdot \frac{\pi}{180^\circ} = -9 \cdot \frac{\pi}{4} = \boxed{-\frac{9\pi}{4}}$$

4. Find the exact value of the given trigonometric expression.

(a) $\sin -5\pi/6$ $\boxed{-\frac{1}{2}}$

(b) $\cos 120^\circ$ $\boxed{-\frac{1}{2}}$

(c) $\tan -60^\circ$ $\boxed{-\sqrt{3}}$

(d) $\csc -25\pi/4$ $\boxed{\sqrt{2}}$

(e) $\sec 150^\circ$ $\boxed{-\frac{2}{\sqrt{3}} \text{ OR } -\frac{2\sqrt{3}}{3}}$

(f) $\cot 3\pi/2$ $\boxed{0}$

5. Recall the following Pythagorean identities.

$$\sin^2 x + \cos^2 x = 1, \qquad \tan^2 x + 1 = \sec^2 x, \qquad 1 + \cot^2 x = \csc^2 x$$

(a) Write $\sin t$ in terms of $\cos t$, where t terminate in quadrant III.

$$\sin^2 t = 1 - \cos^2 t$$

$$\sin t = \pm\sqrt{1 - \cos^2 t}$$

SINCE $\sin t < 0$ IN Q III, $\boxed{\sin t = -\sqrt{1 - \cos^2 t}}$

(b) Write $\cos t$ in terms of $\tan t$, where t terminate in quadrant IV.

$$\sec^2 t = \tan^2 t + 1$$

$$\cos^2 t = \frac{1}{\tan^2 t + 1}$$

$$\cos t = \pm\frac{1}{\sqrt{\tan^2 t + 1}}$$

SINCE $\cos t > 0$ IN Q IV

$$\boxed{\cos t = \frac{1}{\sqrt{\tan^2 t + 1}}}$$

6. Solve each of the following systems of equations. If no solution exists, write *no solution.* If there are an infinite number of solutions, enter the general solution in terms of t, where t is any real number.

(a) $\begin{cases} 2x - 6y &= 10 \quad (1) \\ -3x + 9y &= -15 \quad (2) \end{cases}$

$3(1) + 2(2) : \quad 0 = 0 \quad \Rightarrow \quad \infty$ SOLUTIONS

IF $x = t$ THEN $2t - 6y = 10$

$-6y = 10 - 2t$

$y = \frac{2t - 10}{6} = \frac{t-5}{3}$

$$\boxed{\left(t, \frac{t-5}{3} \right), \ t \in \mathbb{R}}$$

(b) $\begin{cases} \frac{3}{2}x - \frac{1}{3}y &= \frac{1}{2} \quad (1) \\ 2x - \frac{1}{2}y &= -\frac{1}{2} \quad (2) \end{cases}$

$4(1) - 3(2) : \quad -\frac{4}{3}y + \frac{3}{2}y = 2 + \frac{3}{2}$

$\frac{1}{6}y = \frac{7}{2}$

$y = 21$

THEN $2x = -\frac{1}{2} + \frac{1}{2}y = -\frac{1}{2} + \frac{21}{2} = 10$ $\quad \boxed{(5, 21)}$

$x = 5$

7. Solve each of the following systems of equations. If no solution exists, write *no solution.* If there are an infinite number of solutions, enter the general solution in terms of t, where t is any real number.

(a) $$\begin{cases} x & & -4z & = 1 & (1) \\ 2x & -y & -6z & = 4 & (2) \\ 2x & +3y & -2z & = 8 & (3) \end{cases}$$

(2) -2(1) : $\quad -y + 2z = 2 \quad \boxed{1}$

(3) -2(1) : $\quad 3y + 6z = 6 \quad \boxed{2}$

$\boxed{2} + 3\boxed{1}$: $\quad 12z = 12$

$z = 1$

THEN $y = 0$

THEN $X = 5$

$\boxed{(5,0,1)}$

(b) $$\begin{cases} x & +2y & -z & = 1 & (1) \\ 2x & +3y & -4z & = -3 & (2) \\ 3x & +6y & -3z & = 4 & (3) \end{cases}$$

(3) -3(1) : $\quad 0 = 1 \quad$ FALSE!

$\boxed{\text{NO SOLUTION}}$