

Name: _____
Math 195 Precalculus

12/21/2015
Final Exam

This exam has 4 parts, each containing 6 questions. **You must answer 5 questions and omit 1 question from each part.** Please indicate the questions you wish to omit by crossing them out with a large "X". Each question not omitted is worth 5 points, for a total of 100 points.

Please show all work and **box your final answers.** Calculators are not allowed, and cellphones should be put away.

Part I

1. Let $f(x) = 3x^2 - 2x + 1$. Find and simplify $\frac{f(x+h) - f(x)}{h}$.

2. Rewrite the following expression as a simplified sum

$$(x + x^{1/3})(x + x^{2/3}) + (2 - x)(x - x^{1/3})$$

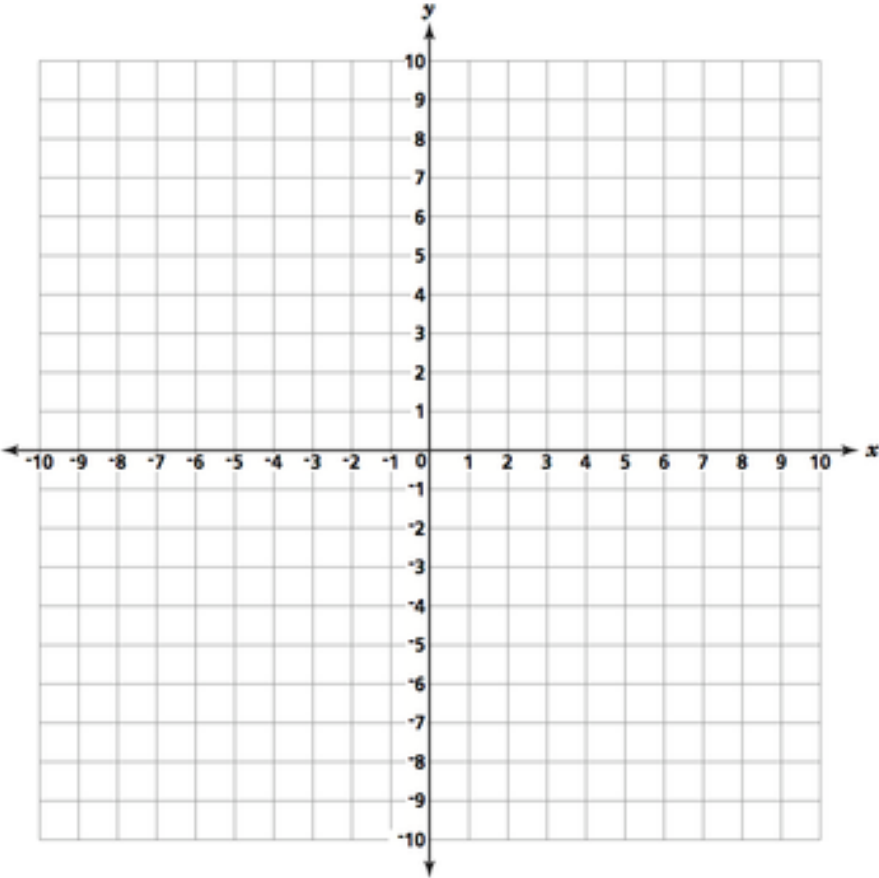
3. Given the one-to-one function $f(x) = \frac{2}{3x - 2}$, find $f^{-1}(x)$.

4. Solve the following inequality and write the solution using interval notation.

$$5x^2 + 3x \geq 3x^2 + 2$$

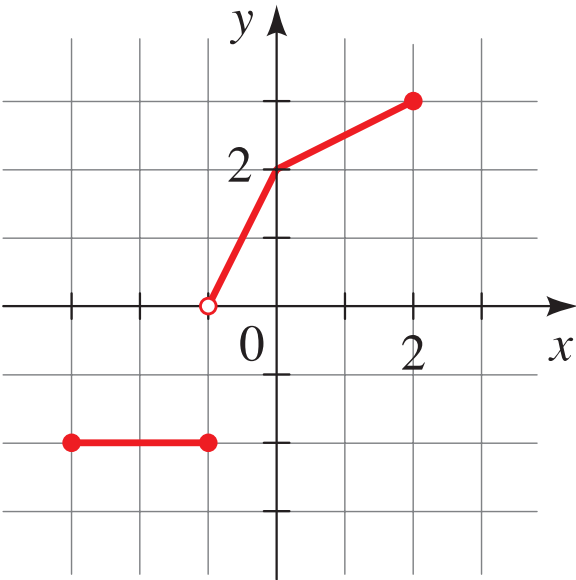
5. Give an equation of the line passing through the point $(1, 7)$ that is perpendicular to the line passing through the points $(2, 5)$ and $(-1, 3)$.

6. In the xy -plane below, sketch the graph $y = 4x^2 - 16x + 7$. Include the coordinates of all intercepts and the vertex.



Part II

7. Use interval notation to specify the domain and range of the function whose graph is given below.



8. Factor the following expression completely.

$$6(x - 2)^{1/3} - 3x(x - 2)^{-2/3}$$

9. Let the functions f and g be defined as follows.

$$f(x) = \frac{1}{x^2 + 1}, \quad g(x) = \frac{1}{x} - 1$$

(a) Find $f(g(x))$.

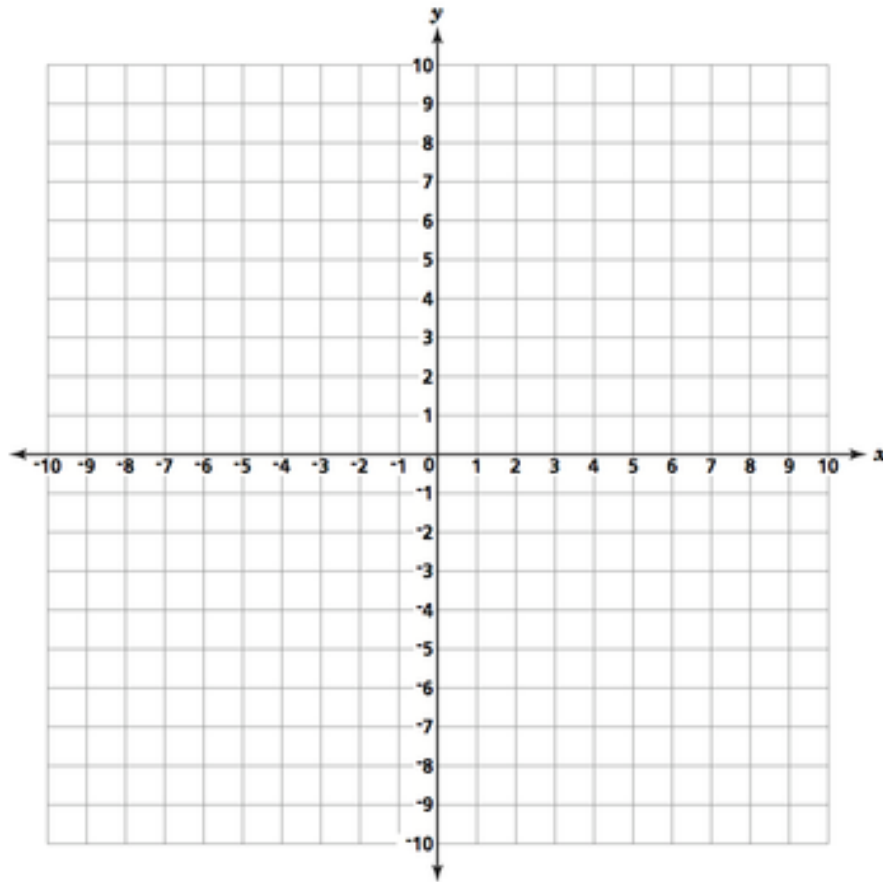
(b) Find $g(f(-4))$.

10. (a) Convert $\frac{5\pi}{12}$ radians to degrees.

(b) Use the addition formula for cosine to find $\cos \frac{5\pi}{12}$.

Note that you may work in degrees, using your answer to part (a), if you prefer.

11. In the xy -plane below, sketch the graph $y = x(x + 5)^4(x - 3)^3(x - 7)^2$. Be sure to clearly show the *end behavior* of the graph, as well as where the graph *crosses* the x -axis and where the graph *bounces off* the x -axis.



12. Suppose the population of a particular colony of ants is found to be 650 on January 1, 2010, and 900 on January 1, 2011. Assume the population of the colony grows exponentially.

(a) Find a function $P(t)$ that gives the population of the ant colony t years after January 1, 2010.

(b) How many years after January 1, 2010 will the population of the ant colony reach 3000?

Part III

13. Solve the following logarithmic equation.

$$\log_5(x + 1) - \log_5(x - 1) = 2$$

14. Find *all* solutions to the following trigonometric equation.

$$4 \sin^2 \theta - 3 = 0$$

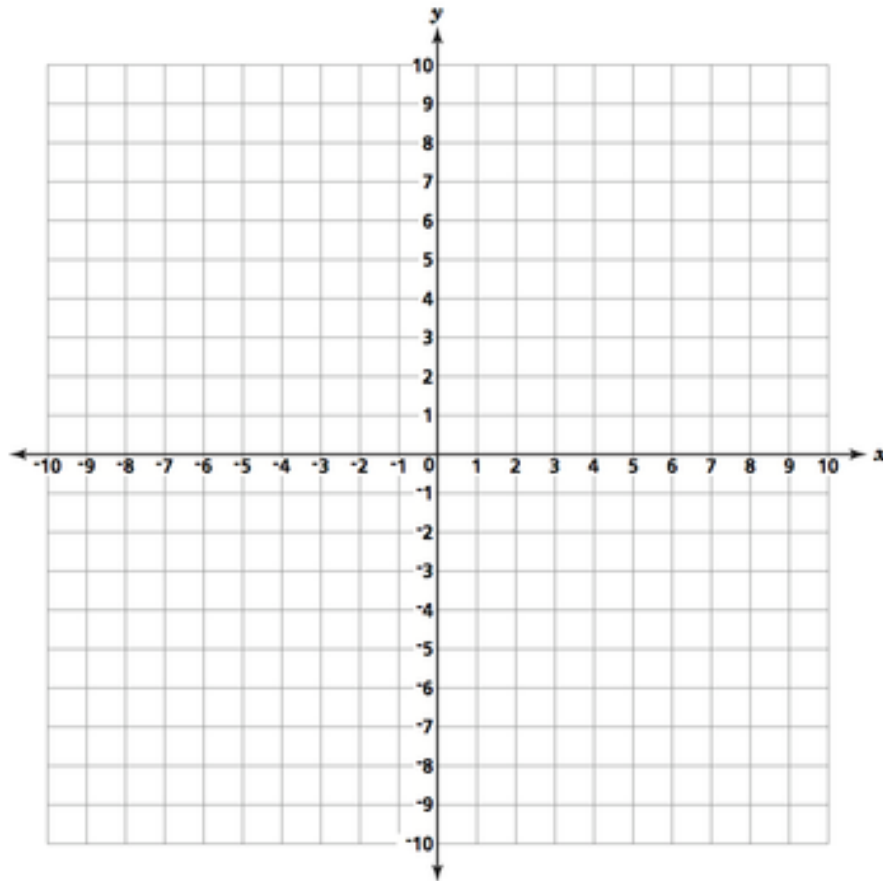
15. Add the following rational expressions and simplify the sum.

$$\frac{3}{x^2 + 3x} + \frac{1}{x^2 + 7x + 12} - \frac{3}{x^2 + 4x}$$

16. Rewrite the following expression as a simplified sum.

$$(x - 5)^2 - (x + 3)(2x - 1) + 2x^2$$

17. In the xy -plane below, sketch the graph $y = 2^{x+1} - 4$ and label all intercepts. Also sketch and give an equation for any asymptotes.



18. Simplify the following rational expression.

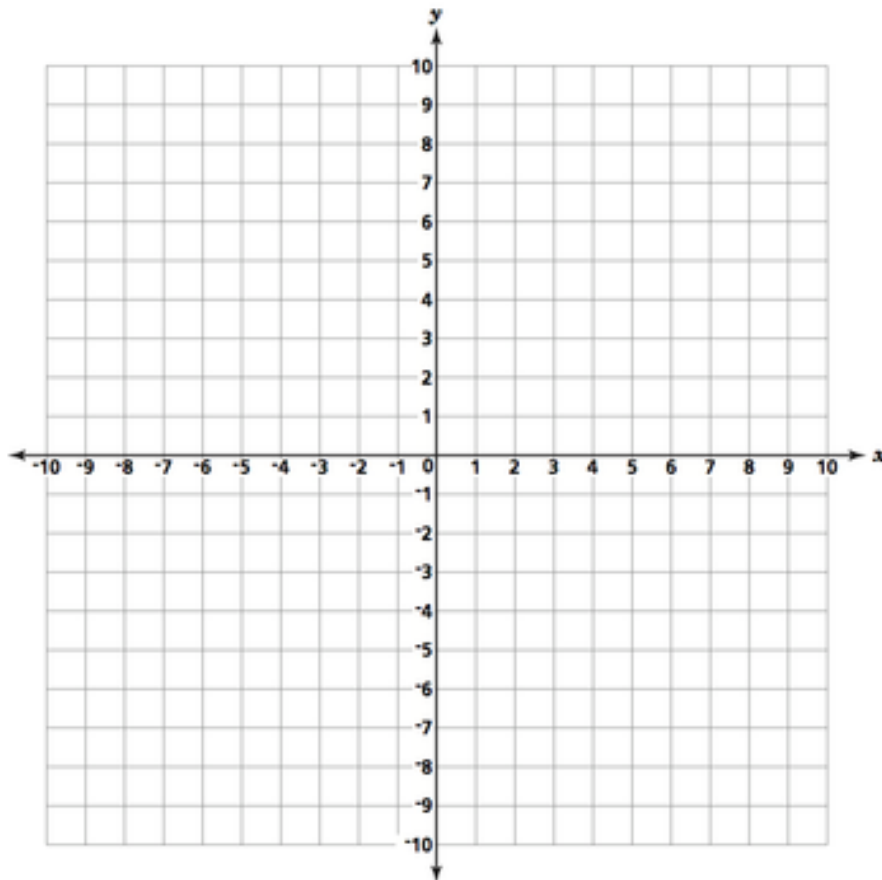
$$\frac{x^3 - x^2 - 4x + 4}{x^2 - 3x + 2}$$

Part IV

19. (a) Find $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

(b) Verify the trigonometric identity $(1 - \cos^2 \theta)(1 + \cot^2 \theta) = 1$.

20. In the xy -plane below, sketch one period of the graph $y = 2 \cos\left(\frac{x}{2}\right) + 1$. Include the coordinates of all intercepts, local maximums, and local minimums.



21. Use interval notation to describe the solution set of the following inequality.

$$\left| \frac{1}{4}x - 2 \right| \geq 3$$

22. Solve the following exponential equation.

$$1 + e^{4x+1} = 20$$

23. Use interval notation to describe the domain of the domain of the following function.

$$f(x) = \frac{x^2 - 25}{\sqrt{x^2 - 4}}$$

24. Find the center and radius of the circle described by the following equation.

$$x^2 + y^2 + 4x + 12 = 6y$$