Name: *ANSWER KEY* 11/4/2015

Math 195 Precalculus **Midterm Exam**

Please show all work and **box your final answers**. Calculators are not allowed and cellphones should be put away. Good luck!

1. Simplify the following expressions and eliminate any negative exponents. Assume all letters denote positive numbers.

(a) (4 points) $\left(\frac{2a^{-1}b}{a^2b^{-3}}\right)^{-3}$ $= \dfrac{2^{-3}(a^{-1})^{-3}b^{-3}}{(a^2)^{-3}(b^{-3})^{-3}} = \dfrac{2^{-3}a^{3}b^{-3}}{a^{-6}b^{9}}$

$$= \frac{a^3a^6}{2^3b^3b^9} = \boxed{\frac{a^9}{8b^{12}}}$$

(b) (4 points) $\frac{(32x^5y^{-3/2})^{2/5}}{(x^{5/3}y^{2/3})^{3/5}}$ $= \dfrac{32^{2/5}(x^5)^{2/5}(y^{-3/2})^{2/5}}{(x^{5/3})^{3/5}(y^{2/3})^{3/5}}$

$$= \frac{(32^{1/5})^2x^2y^{-3/5}}{xy^{2/5}} = \frac{2^2x^2}{xy^{2/5}y^{3/5}} = \boxed{\frac{4x}{y}}$$

(c) (4 points) $\frac{2(1+x)^{1/2}-x(1+x)^{-1/2}}{1+x}$ $= \dfrac{(1+x)^{-1/2}[2(1+x)-x]}{(1+x)}$

$$= \frac{2+2x-x}{(1+x)^{1/2}(1+x)} = \boxed{\frac{2+x}{(1+x)^{3/2}}}$$

2. Solve the following inequalities. Use interval notation to express your answers.

(a) (8 points) $|x+5| > 2$

EITHER $x + 5 > 2$ OR $x + 5 < -2$

$x > -3$ OR $x < -7$

$$\boxed{(-\infty, -7) \cup (-3, \infty)}$$

(b) (8 points) $5x^2 + 3x \geq 3x^2 + 2$

$$2x^2 + 3x - 2 \geq 0$$

ZEROS: $2x^2 + 3x - 2 = 0$

$(2x - 1)(x + 2) = 0$

$x = -2, \frac{1}{2}$

		-2		$\frac{1}{2}$	
$(2x-1)$	$-$		$-$		$+$
$(x+2)$	$-$		$+$		$+$
$2x^2+3x-2$	$+$		$-$		$+$
	✓				✓

$$\boxed{(-\infty, -2] \cup [\tfrac{1}{2}, \infty)}$$

3. (8 points) Find the center and radius of the circle described by the following equation.

$$\frac{1}{3}\left[3x^2 + 3y^2 + 12x + 36 = 18y\right]$$

$x^2 + 4x + y^2 - 6y = -12$

$x^2 + 4x + 4 + y^2 - 6y + 9 = -12 + 4 + 9$

$(x+2)^2 + (y-3)^2 = 1$

$$\boxed{\begin{array}{ll} \text{CENTER :} & (-2, 3) \\ \text{RADIUS :} & 1 \end{array}}$$

4. (8 points) Give an equation for the line through the point $(-1, 2)$ that is parallel to the line joining points $(-2, 2)$ and $(1, -2)$.

$$m = \frac{(-2)-(2)}{(1)-(-2)} = \frac{-4}{3}$$

POINT - SLOPE FORMULA : $\boxed{y - 2 = -\frac{4}{3}(x+1)}$, OR

SLOPE - INTERCEPT FORMULA : $\boxed{y = -\frac{4}{3}x + \frac{2}{3}}$

5. (8 points) Give an equation for the line perpendicular to $6y - 2x = 5$ that passes through the point $(-4, 7)$.

$$6y = 2x + 5$$

$$y = \frac{1}{3}x + \frac{5}{6}$$

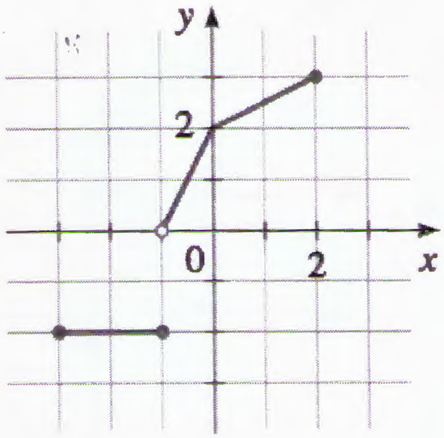
↑

$\perp$ SLOPE : -3

POINT-SLOPE FORMULA : $\boxed{y - 7 = -3(x + 4)}$, OR

SLOPE-INTERCEPT FORMULA : $\boxed{y = -3x - 5}$

6. (8 points) Determine if the following graph is the graph of a function. If it is not, explain why. If it is, explain why and find its domain and range.



YES, IT IS A FUNCTION BECAUSE IT PASSES THE HORIZONTAL LINE TEST.

DOMAIN : $[-3, 2]$

RANGE : $\{-2\} \cup (0, 3]$

7. (8 points) Find the average rate of change of $f(z) = 1 - 3z^2$ between $z = -2$ and $z = 0$.

$$\frac{f(0) - f(-2)}{(0) - (-2)} = \frac{(1 - 3(0)^2) - (1 - 3(-2)^2)}{2}$$

$$= \frac{1 - 1 + 12}{2} = \boxed{6}$$

8. (8 points) Let f be the one-to-one function $f(x) = \dfrac{3 - 4x}{8x - 1}$. Find $f^{-1}(x)$ and the range of f.
Hint: The range of f is equal to the domain of f^{-1}.

$$y = \frac{3 - 4x}{8x - 1} \longrightarrow y(8x - 1) = 3 - 4x \longrightarrow 8xy - y = 3 - 4x$$

$$\longrightarrow 8xy + 4x = 3 + y \longrightarrow (8y + 4)x = 3 + y$$

$$\longrightarrow x = \frac{3 + y}{8y + 4}$$

$$\therefore \boxed{f^{-1}(x) = \frac{3 + x}{8x + 4}}$$

RANGE(f) = DOMAIN$(f^{-1}) = \{x : 8x + 4 \neq 0\}$

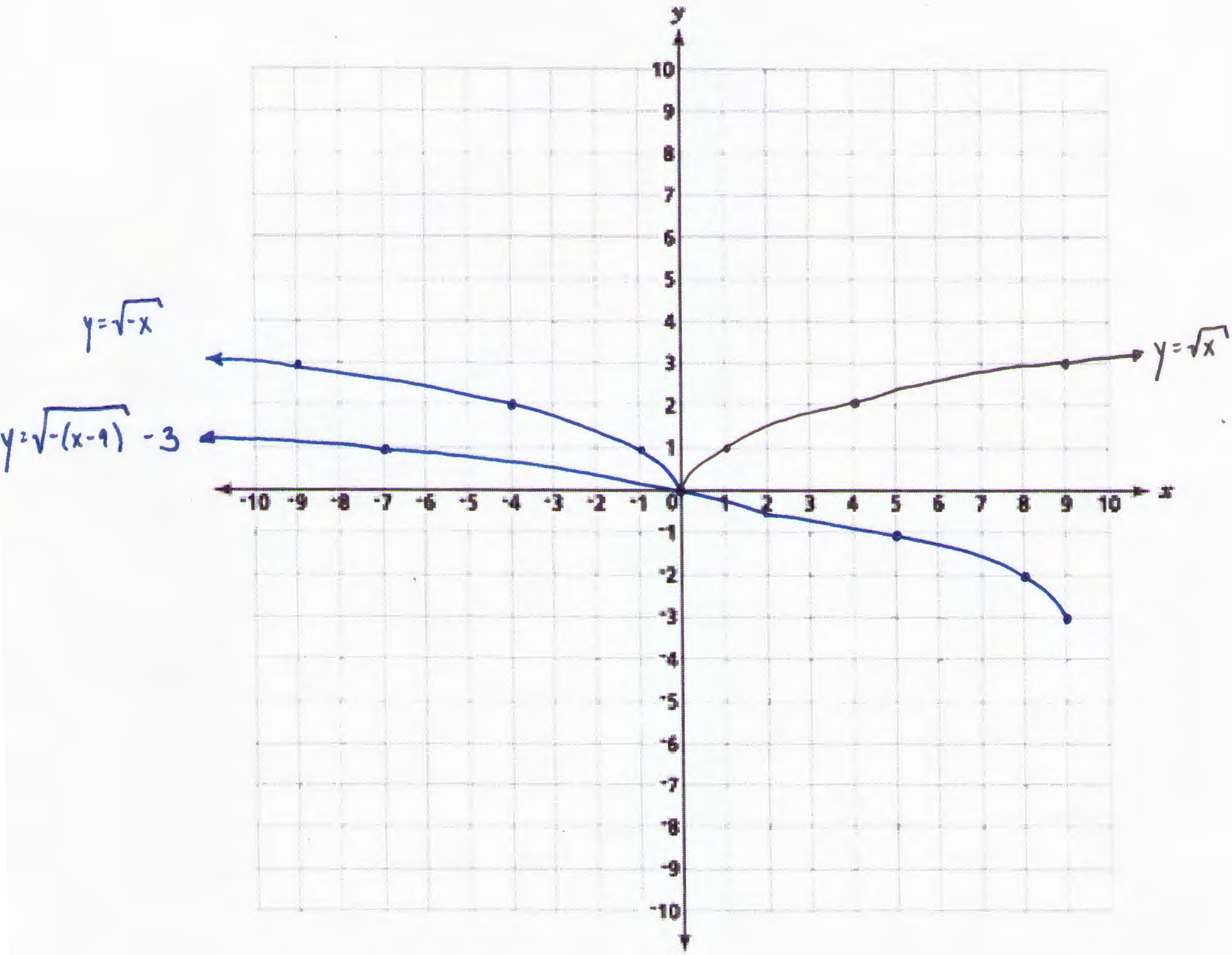
$$\boxed{= \{x : x \neq -\tfrac{1}{2}\}, \text{ or } = (-\infty, -\tfrac{1}{2}) \cup (-\tfrac{1}{2}, \infty)}$$

9. In the space below, sketch and label the graphs of the following equations.

(a) (4 points) $y = \sqrt{x}$

(b) (4 points) $y = \sqrt{-x}$

(c) (4 points) $y = \sqrt{-(x-9)} - 3$



10. Let

$$f(x) = \frac{1}{x}, \qquad g(x) = x^3, \qquad h(x) = x^2 + 2.$$

Evaluate the following expressions. For parts (b) and (c), do not bother trying to simplify/expand your answer, just leave it how you found it.

(a) (4 points) $f \circ h(3)$, in other words $f(h(3))$

$h(3) = 3^2 + 2 = 9 + 2 = 11$

$f(h(3)) = f(11) = \boxed{\frac{1}{11}}$

(b) (4 points) $f \circ g \circ h(a+b)$, in other words, $f(g(h(a+b)))$

$$\boxed{\frac{1}{\left((a+b)^2 + 2\right)^3}}$$

(c) (4 points) $g \circ h \circ f \circ h(x)$, in other words $g(h(f(h(x))))$

$$\boxed{\left(\left(\frac{1}{x^2+2}\right)^2 + 2\right)^3}$$

11. Consider the quadratic function

$$f(x) = -4x^2 - 12x + 1.$$

(a) (4 points) Express f in standard form, that is $y = a(x-h)^2 + k$.

$$f(x) = -4(x^2 + 3x) + 1$$

$$= -4\left(x^2 + 3x + \frac{9}{4}\right) + 1 + 9$$

$$\boxed{f(x) = -4\left(x + \frac{3}{2}\right)^2 + 10}$$

(b) (4 points) Find the minimum/maximum value of f, say whether it is a minimum or maximum, and then give the range of f.

$$\boxed{\begin{array}{l} \text{MAX VALUE : } 10 \\ \text{RANGE : } (-\infty, 10] \end{array}}$$

(VERTEX : $\left(-\frac{3}{2}, 10\right)$, GRAPH OPENS DOWNWARD)

(c) (4 points) Does the graph of $y = f(x)$ intersect the x-axis? If so, give the x-intercept(s) in simplified form.

$$-4\left(x + \frac{3}{2}\right)^2 + 10 = 0$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{-10}{-4} = \frac{5}{2}$$

$$x + \frac{3}{2} = \pm\sqrt{\frac{5}{2}}$$

$$\boxed{x = -\frac{3}{2} \pm \sqrt{\frac{5}{2}}}$$

OR

OR USE QUAD. FORMULA

$$x = \frac{12 \pm \sqrt{12^2 - 4(-4)(1)}}{2(-4)}$$

$$= \frac{12 \pm \sqrt{160}}{-8}$$

$$= \frac{12 \pm 4\sqrt{10}}{-8}$$

$$\boxed{x = \frac{3 \pm \sqrt{10}}{-2}}$$