

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed and cellphones should be put away. Good luck!

1. (8 points) Rewrite the following expression as one simplified fraction.

$$\frac{1}{x^2 - 4} + \frac{2}{x^2 + 2x}$$

$$= \frac{1}{(x+2)(x-2)} + \frac{2}{x(x+2)} \quad \text{LCD} = x(x+2)(x-2)$$

$$= \frac{x}{x(x+2)(x-2)} + \frac{2(x-2)}{x(x+2)(x-2)} = \frac{x + 2x - 4}{x(x+2)(x-2)}$$

$$= \boxed{\frac{3x - 4}{x(x+2)(x-2)}}$$

2. (8 points) Find the domain of the expression

$$\frac{x - 1}{x^2 - 5x - 24}$$

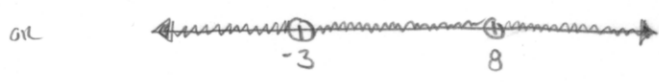
ALL REAL NUMBERS EXCEPT WHERE DENOMINATOR IS 0.

$$x^2 - 5x - 24 = 0$$

$$(x - 8)(x + 3) = 0$$

$$x = 8, \quad x = -3$$

$$\{x \in \mathbb{R}; x \neq 8, x \neq -3\} \quad \text{or} \quad (-\infty, -3) \cup (-3, 8) \cup (8, \infty)$$



3. Find the solution sets of the following inequalities.

(a) (8 points)  $2x^3 + 9 > x^2 + 18x$

$$\underbrace{2x^3 - x^2}_{\phantom{2x^3 - x^2}} - \underbrace{18x + 9}_{\phantom{18x + 9}} > 0$$

Factor by Grouping:

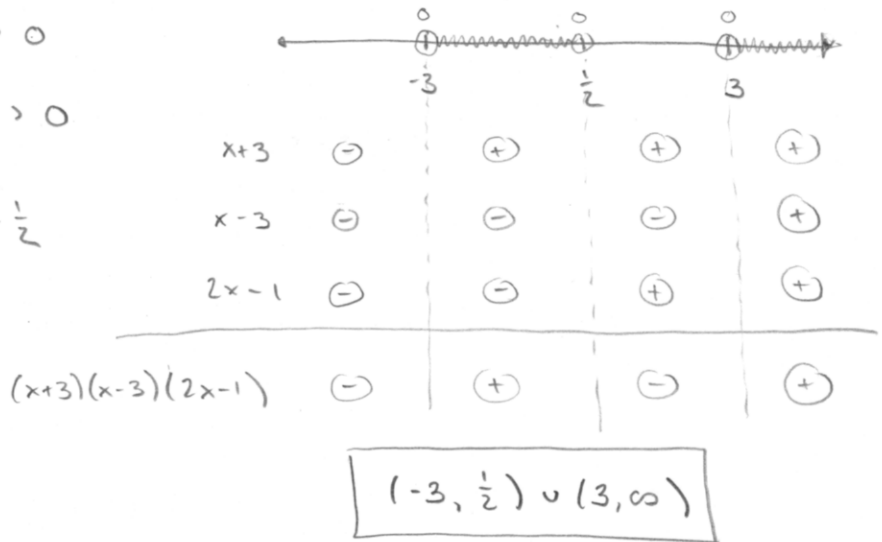
$$x^2(2x-1) - 9(2x-1) > 0$$

$$(x^2 - 9)(2x - 1) > 0$$

$$(x+3)(x-3)(2x-1) > 0$$

Zeros:

$$x = -3 \quad x = 3 \quad x = \frac{1}{2}$$



(b) (8 points)  $|x + 5| > 2$

MEANS EITHER

$$x + 5 < -2$$

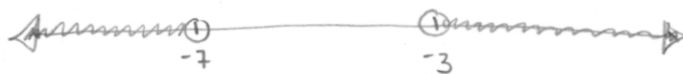
OR

$$x + 5 > 2$$

$$x < -7$$

OR

$$x > -3$$



$(-\infty, -7) \cup (-3, \infty)$

4. (a) (8 points) Give an equation of the circle that has its center at  $(5, -2)$  and passes through the point  $(2, -6)$ .

USE DISTANCE FORMULA TO FIND  $r = \sqrt{(5-2)^2 + (-2+6)^2} = \sqrt{3^2 + 4^2} = 5$

THEN PLUG IN TO CIRCLE EQ:  $(x-h)^2 + (y-k)^2 = r^2$

$$(x-5)^2 + (y+2)^2 = 25$$

- (b) (8 points) Find the center and radius of the circle described by the following equation.

$$x^2 + y^2 + 10y + 31 = 6x$$

$$x^2 - 6x + y^2 + 10y = -31$$

$$(x-3)^2 + (y+5)^2 = -31 + 9 + 25 = 3$$

$\therefore$

$$\begin{array}{l} \text{center } (3, -5) \\ \text{RADIUS} = \sqrt{3} \end{array}$$

5. Let  $l$  be the line described by  $8x - 5y = 1$ .

(a) (8 points) Find the  $x$  and  $y$  intercepts of  $l$ .

$$\begin{aligned} x\text{-int: set } y=0 : 8x &= 1 \\ x &= \frac{1}{8} \rightarrow \boxed{\left(\frac{1}{8}, 0\right) \text{ } x\text{-int}} \end{aligned}$$

$$\begin{aligned} y\text{-int: set } x=0 : -5y &= 1 \\ y &= -\frac{1}{5} \rightarrow \boxed{\left(0, -\frac{1}{5}\right) \text{ } y\text{-int}} \end{aligned}$$

(b) (8 points) Find an equation of the line perpendicular to  $l$  that passes through the point  $(-1, 2)$ .

$$l: 8x - 5y = 1$$

$$-5y = -8x + 1$$

$$y = \boxed{\frac{8}{5}}x - \frac{1}{5}$$

$$\uparrow \text{ NEG. RECIP. } = -\frac{5}{8} \leftarrow \underline{\underline{\text{SLURP}}}$$

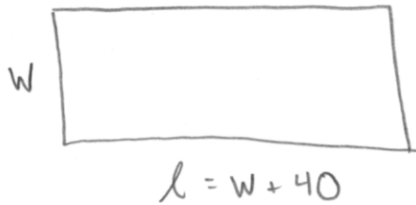
$\uparrow$   
POINT

USING POINT-SLOPE FORMULA:

$$\boxed{y - 2 = -\frac{5}{8}(x + 1)} \quad \text{or} \quad \boxed{y = -\frac{5}{8}x + \frac{11}{8}}$$

$$\text{or } 5x + 8y = 11$$

6. (8 points) Suppose a rectangular garden is to be constructed such that its length is 40 ft longer than its width, and its area is  $500 \text{ ft}^2$ . Find the dimensions of the garden.



$$A = lw$$

$$500 = (w+40)w = w^2 + 40w$$

$$0 = w^2 + 40w - 500$$

$$0 = (w+50)(w-10)$$

✓  
 $w = -50$

↘  
 $w = 10$

reject

(width cannot  
be neg.)

✓

$$w = 10$$

$$l = w + 40 = 10 + 40 = 50$$

