

Directions Answer all questions in the space provided. Show all work and box your final answers. Answers with no work shown will not receive full credit. Answers do not need to be simplified unless specifically stated. Good luck!

1. (10 points) Find the domain of $f(x) = \frac{\sqrt{x-1}}{5x^3 - 35x^2 + 60x}$.

SQUARE ROOT : $x - 1 \geq 0$

$x \geq 1$

Denominator : $5x^3 - 35x^2 + 60 \neq 0$

$5x(x^2 - 7x + 12) \neq 0$

$5x(x-4)(x-3) \neq 0$

$x \neq 0$ $x \neq 4$ $x \neq 3$

COMBINE (INTERSECT)

$x \geq 1$ AND $x \neq 3, 4$

i.e. $[1, 3) \cup (3, 4) \cup (4, \infty)$

2. (10 points) Let $f(x) = x^2 + 2x + 1$. Find and simplify $\frac{f(a+h) - f(a)}{h}$.

$$= \frac{\overbrace{(a+h)^2 + 2(a+h) + 1}^{f(a+h)} - \overbrace{(a^2 + 2a + 1)}^{f(a)}}{h}$$

$$= \frac{a^2 + 2ah + h^2 + 2a + 2h + 1 - a^2 - 2a - 1}{h}$$

$$= \frac{2ah + h^2 + 2h}{h} = \frac{h(2a + h + 2)}{h} = \boxed{2a + h + 2}$$

3. (10 points) Find the average rate of change of $g(t) = 2t^2 - t$ between $t = 3$ and $t = 6$.

$$\frac{g(6) - g(3)}{6 - 3} = \frac{2(6)^2 - (6) - (2(3)^2 - (3))}{6 - 3}$$

$$= \frac{72 - 6 - 18 + 3}{3} = 24 - 2 - 6 + 1 = \boxed{17}$$

4. Let f and g be defined as follows.

$$f(x) = \frac{x}{x+1}, \quad g(x) = \frac{1}{x}$$

(a) (7 points) Find $(f \circ g)(x)$ and state its domain.

$$f(g(x)) = f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}}{\frac{1}{x} + 1} \quad \text{or} \quad \frac{1}{1+x}$$

DOMAIN: $x \neq 0$ AND $\frac{1}{x} + 1 \neq 0$ $x \neq -1, 0$
 $x \neq -1$ i.e. $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

(b) (7 points) Find $(f \circ f)(x)$ and state its domain.

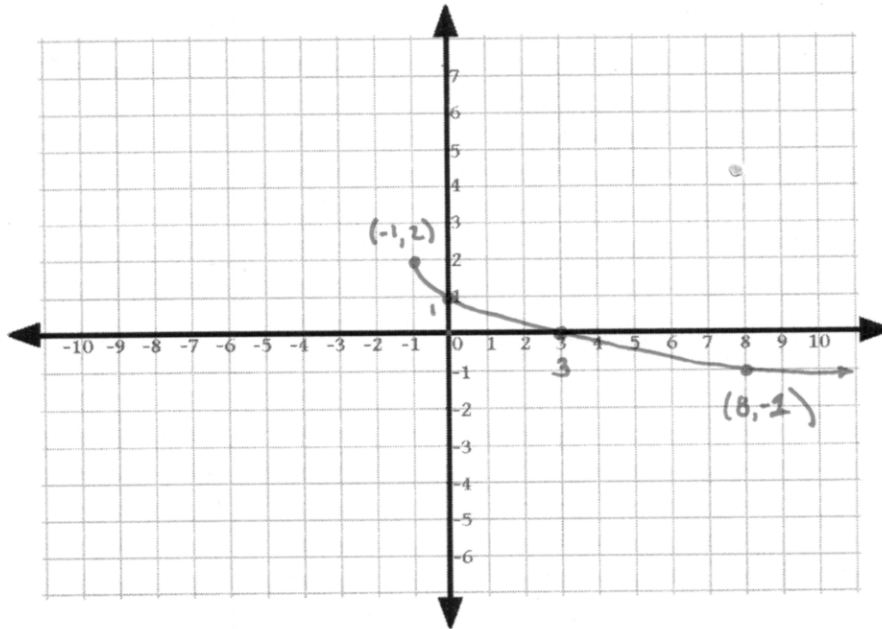
$$f(f(x)) = f\left(\frac{x}{x+1}\right) = \frac{\frac{x}{x+1}}{\frac{x}{x+1} + 1} \quad \text{or} \quad \frac{x}{2x+1}$$

DOMAIN: $x+1 \neq 0$ AND $\frac{x}{x+1} + 1 \neq 0$
 $x \neq -1$ $x \neq -\frac{1}{2}$

$$x \neq -1, -\frac{1}{2}$$

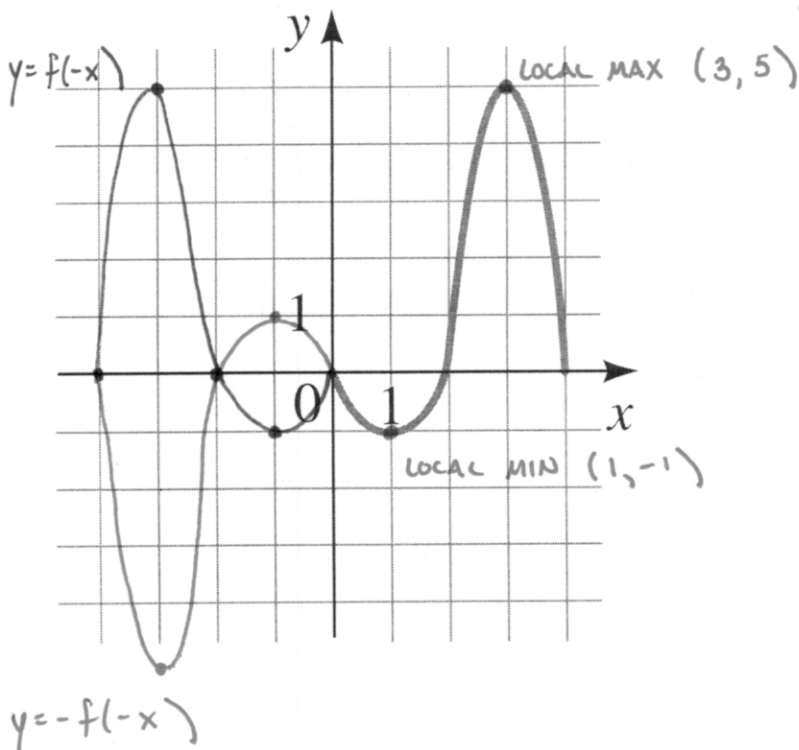
i.e. $(-\infty, -1) \cup (-1, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

5. (10 points) Sketch the graph of $f(x) = 2 - \sqrt{x+1}$ on the coordinate plane below. Label all x -intercepts and y -intercepts, and two additional points on the graph.



6. The graph of $f(x)$ is shown below.

- (a) (7 points) Label all local maximums and minimums of f with their coordinates.
 (b) (7 points) On the same coordinate plane, sketch and label the graphs $y = f(-x)$ and $y = -f(-x)$.



7. (10 points) Let f be the one-to-one function $f(x) = \frac{3-4x}{8x-1}$. Find $f^{-1}(x)$.

$$y = \frac{3-4x}{8x-1} \quad \text{Solve for } x : \quad 8xy - y = 3 - 4x$$
$$x(8y+4) = 3+y$$
$$x = \frac{3+y}{8y+4}$$

$$f^{-1}(x) = \frac{3+x}{8x+4}$$

8. (10 points) Find the maximum or minimum value of the function $q(x) = 24x + 100 - 6x^2$.

$$q(x) = -6x^2 + 24x + 100$$

$$a = -6 \quad b = 24 \quad c = 100$$

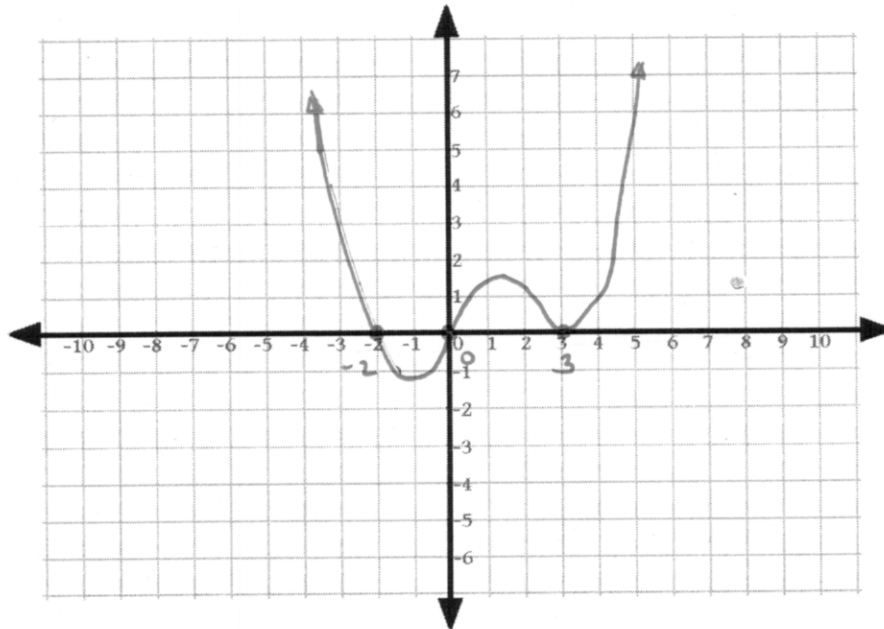
$a < 0$ so MAX VAL of q is

$$q\left(-\frac{b}{2a}\right) = q\left(-\frac{24}{2(-6)}\right) = q(2) = 24(2) + 100 - 6(2)^2$$

$$= 48 + 100 - 24$$

$$= \boxed{124 \text{ MAX VAL}}$$

9. (10 points) Sketch the graph of $P(x) = x^3(x+2)(x-3)^2$ on the coordinate plane below. Clearly show the end behavior of the graph, and label all x -intercepts and y -intercepts.



Factor	zero	MULTIPLICITY	X-AXIS BEHAVIOR
x^3	0	3	CROSS
$x+2$	-2	1	CROSS
$(x-3)^2$	3	2	BOUNCE

+

6TH DEGREE POLY (EVEN)

LEAD COEFF > 0



y-int: $P(0) = 0$.