Name: *ANSWER KEY* 7/27/2016

Math 195-1XD Final Exam

Directions Answer all questions in the space provided. Show all work and box your final answers. Answers with no work shown will not receive full credit. Answers do not need to be simplified unless specifically stated. Good luck!

1. (a) Simplify the following expression and eliminate any negative exponents.

$$\left(\frac{2a^{-1}b}{a^2b^{-3}}\right)^{-3}$$

$$= \frac{2^{-3}a^{3}b^{-3}}{a^{-6}b^{9}} = \frac{a^3a^6}{2^3b^3b^9} = \boxed{\frac{a^9}{8b^{12}}}$$

(b) Perform the division and simplify.

$$\frac{x+3}{4x^2-9} \div \frac{x^2+7x+12}{2x^2+7x-15}$$

$$= \frac{\cancel{(x+3)}}{(2x+3)\cancel{(2x-3)}} \cdot \frac{\cancel{(2x-3)}(x+5)}{\cancel{(x+3)}(x+4)}$$

$$= \boxed{\frac{x+5}{(2x+3)(x+4)} \quad \text{OR} \quad \frac{x+5}{2x^2+11x+12}}$$

(c) Perform the addition/subtraction and simplify.

$$\frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{3}{x^2-1}$$

LCD = $(x+1)^2(x-1)$ $(x+1)(x-1)$

$$= \frac{(x+1)(x-1)}{(x+1)^2(x-1)} - \frac{2(x-1)}{(x+1)^2(x-1)} + \frac{3(x+1)}{(x+1)^2(x-1)}$$

$$= \frac{x^2-1-2x+2+3x+3}{(x+1)^2(x-1)} = \boxed{\frac{x^2+x+4}{(x+1)^2(x-1)} \quad \text{OR} \quad \frac{x^2+x+4}{x^3+x^2-x-1}}$$

2. (a) Use log laws to expand the expression $\log\left(x\sqrt{\frac{y}{z}}\right)$

$$= \log x + \frac{1}{2}\log\frac{y}{z} = \boxed{\log x + \frac{1}{2}(\log y - \log z)}$$

(b) Compute $\ln\left(\frac{1}{\sqrt{e}}\right)$

$$= \ln e^{-1/2} = \boxed{-\frac{1}{2}}$$

(c) Suppose a 72 gram sample of radioactive material takes 80 years to decay to 60 grams. Assuming exponential decay, how long would it take for the sample to decay to 50 grams?

$$M(t) = 72K^{t}$$

$$60 = M(80) = 72K^{80}$$

$$\frac{60}{72} = K^{80}$$

$$K = \left(\frac{5}{6}\right)^{1/80}$$

$$\therefore M(t) = 72\left(\frac{5}{6}\right)^{t/80}$$

Solve for t:

$$M(t) = 72\left(\frac{5}{6}\right)^{t/80} = 50$$

$$\left(\frac{5}{6}\right)^{t/80} = \frac{25}{36}$$

$$\frac{t}{80}\ln\frac{5}{6} = \ln\frac{25}{36}$$

$$\boxed{t = \frac{80\ln\frac{25}{36}}{\ln\frac{5}{6}}}$$

$$\left(= \frac{80\ln\left(\frac{5}{6}\right)^{2}}{\ln\frac{5}{6}} = \frac{160\ln\frac{5}{6}}{\ln\frac{5}{6}} \boxed{= 160} \right)$$

3. Solve the following equations.

(a) $\dfrac{1}{x-1} + \dfrac{1}{x+2} = \dfrac{5}{4}$ LCD $= 4(x-1)(x+2)$

$4(x+2) + 4(x-1) = 5(x-1)(x+2)$

$4x + 8 + 4x - 4 = 5x^2 + 5x - 10$

$0 = 5x^2 - 3x - 14$

$0 = (5x + 7)(x - 2)$

$\boxed{x = -\frac{7}{5}, \ 2}$

(b) $1 + e^{4x+1} = 20$

$e^{4x+1} = 19$

$4x + 1 = \ln 19$

$4x = \ln 19 - 1$

$\boxed{x = \dfrac{\ln 19 - 1}{4} \quad \text{or} \quad \dfrac{1}{4} \ln \dfrac{19}{e}}$

(c) $\log_2 x + \log_2(x-3) = 2$

$\log_2 (x^2 - 3x) = 2$

$x^2 - 3x = 2^2$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x = 4}$

$x = 4, \quad x = -1$
(REJECT)

4. (a) Find the center and radius of the circle with the following equaltion.

$$x^2 + y^2 - 3x + 8y + \frac{9}{4} = 0$$

$$x^2 - 3x + y^2 + 8y = -\frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 + (y+4)^2 = -\frac{9}{4} + \frac{9}{4} + 16$$

$$\left(x - \frac{3}{2}\right)^2 + (y+4)^2 = 16$$

CENTER : $\left(\frac{3}{2}, -4\right)$

RADIUS : 4

(b) Solve the following inequality and express the solution using interval notation.

$$|3x - 5| \geq 2$$

EITHER $3x - 5 \leq -2$ OR $3x - 5 \geq 2$

$3x \leq 3$ $\qquad$ $3x \geq 7$

$x \leq 1$ $\qquad$ $x \geq \frac{7}{3}$

$$\boxed{(-\infty, 1] \cup \left[\frac{7}{3}, \infty\right)}$$

5. Consider the following functions.

$$f(x) = x^2 - 7x + 11, \qquad g(x) = \frac{1}{x+1}, \qquad s(x) = \sqrt{x-3}$$

(a) Find $\dfrac{f(a+h) - f(a)}{h}$.

$$= \frac{(a+h)^2 - 7(a+h) + 11 - (a^2 - 7a + 11)}{h}$$

$$= \frac{\cancel{a^2} + 2ah + h^2 - \cancel{7a} - 7h + \cancel{11} - \cancel{a^2} + \cancel{7a} - \cancel{11}}{h}$$

$$= \frac{h(2a + h - 7)}{h} = \boxed{2a + h - 7}$$

(b) Find $(g \circ f)(x)$ and state its domain.

$$g(f(x)) = \frac{1}{f(x) + 1} = \boxed{\frac{1}{x^2 - 7x + 12}}$$

DOMAIN: $x^2 - 7x + 12 \neq 0$

$(x-3)(x-4) = 0$

$x \neq 3, 4$

$$\boxed{(-\infty, 3) \cup (3, 4) \cup (4, \infty) \quad \text{or} \quad x \neq 3, 4}$$

(c) Find $(s \circ g)(x)$ and state its domain.

$$s(g(x)) = \sqrt{g(x) - 3} = \boxed{\sqrt{\frac{1}{x+1} - 3}, \quad \text{DOMAIN } \left(-1, -\tfrac{2}{3}\right]}$$

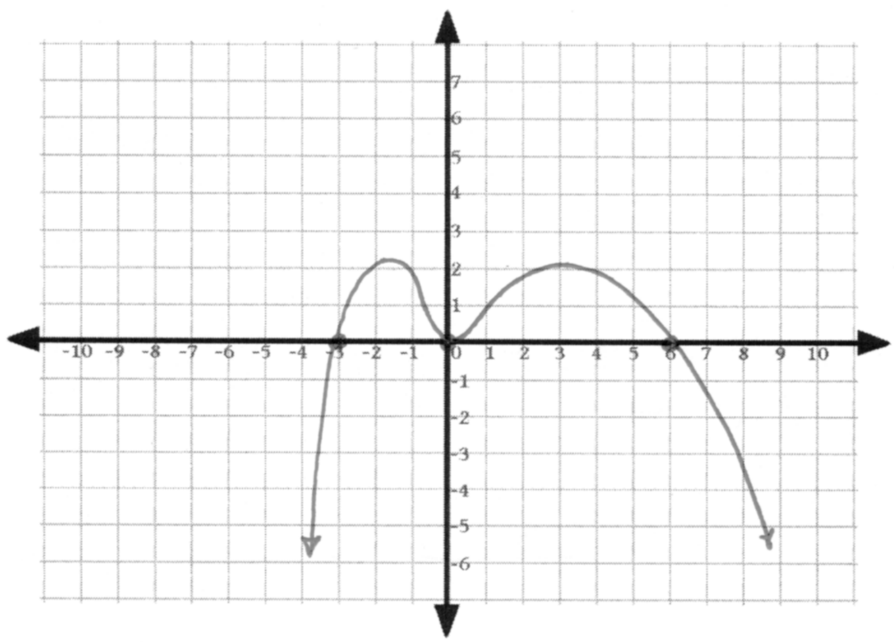
DOMAIN: $\dfrac{1}{x+1} - 3 \geq 0$

$\dfrac{-3x - 2}{x + 1} \geq 0$

		UND. -1		ZERO $-\frac{2}{3}$	
$-3x-2$	+		+		−
$x+1$	−		+		+
$\frac{-3x-2}{x+1}$	−		+		−

6. Sketch the graph of the following polynomial on the coordinate plane below. Label all x-intercepts and y-intercepts. Be sure that it is clear on which intervals the function is positive/negative, and what the end behavior is.

$$p(x) = -x^2(x+3)(x-6)$$



4^{TH} DEGREE, NEG. LEAD COEFF $\Rightarrow$ END BEHAVIOR

X-INTERCEPTS : CROSS X-AXIS @ $x = -3$ (MULTIPLICITY 1 - ODD)

BOUNCE X-AXIS @ $x = 0$ (MULTIPLICITY 2 - EVEN)

CROSS X-AXIS @ $x = 6$ (MULTIPLICITY 1 - ODD)

8. (a) Give an equation for the line that is perpendicular to $4x + 2y = 1$ and passes through the point $(-3, 5)$.

$$2y = -4x + 1$$

$$y = -2x + \frac{1}{2}$$

$$\downarrow$$

$\perp$ SLOPE $= \frac{1}{2}$

PNT. SLOPE FORM: $y - 5 = \frac{1}{2}(x + 3)$, OR

SLOPE-INT FORM: $y = \frac{1}{2}x + \frac{13}{2}$

(b) Given $f(x) = \dfrac{4 - 7x}{1 - 3x}$, find $f^{-1}(x)$.

LET $y = \dfrac{4-7x}{1-3x}$

$$y(1-3x) = 4 - 7x$$

$$y - 3xy = 4 - 7x$$

$$-3xy + 7x = 4 - y$$

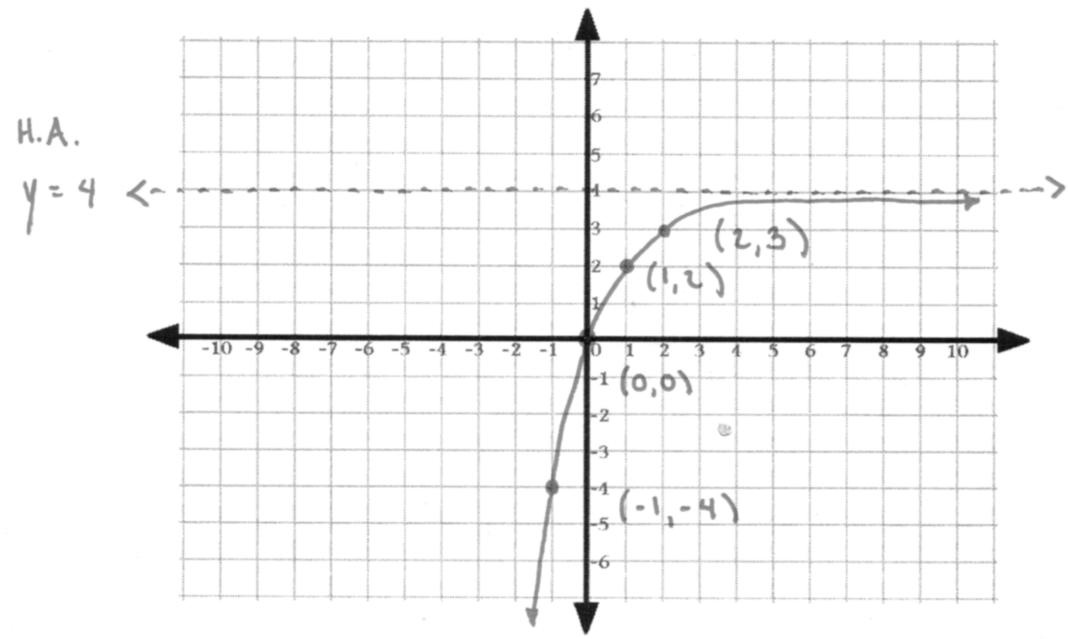
$$x(-3y + 7) = 4 - y$$

$$x = \frac{4-y}{7-3y}$$

$$\boxed{f^{-1}(x) = \frac{4-x}{7-3x}}$$

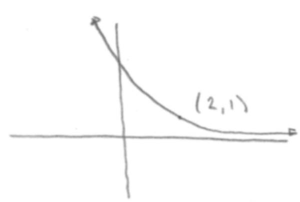
9. Sketch the graph of the following exponential function on the coordinate plane below. Label all x-intercepts and y-intercepts, and two additional points. Label all vertical/horizontal asymptotes with their equations.

$$f(x) = 4 - \left(\frac{1}{2}\right)^{x-2}$$



$y = \left(\frac{1}{2}\right)^{x}$

$y = \left(\frac{1}{2}\right)^{x-2}$

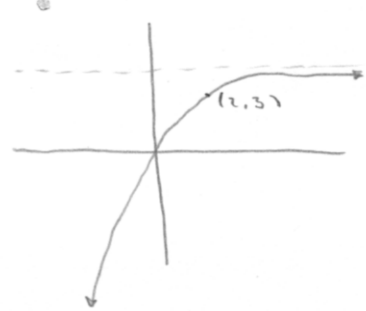


(SHIFT 2 RIGHT)

$y = -\left(\frac{1}{2}\right)^{x-2}$

(REFLECT ACROSS X-AXIS)

$y = 4 - \left(\frac{1}{2}\right)^{x-2}$



(SHIFT UP 4)

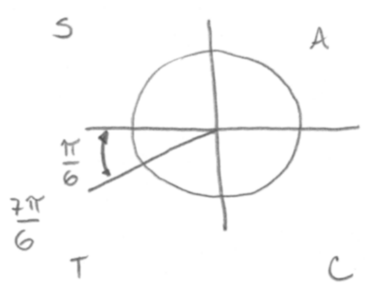
10. (a) Convert 300° to radians.

$$300^\circ \cdot \frac{\pi \text{ RAD}}{180^\circ} = \boxed{\frac{5\pi}{3} \text{ RAD.}}$$

(b) Convert $\dfrac{2\pi}{5}$ radians to degrees.

$$\frac{2\pi}{5} \text{ RAD} \cdot \frac{180^\circ}{\pi \text{ RAD}} = \boxed{72^\circ}$$

(c) Find the exact value of $\cos(7\pi/6)$.



REF. ANGLE $\dfrac{\pi}{6}$

$$\cos\frac{7\pi}{6} = -\cos\frac{\pi}{6} = \boxed{-\frac{\sqrt{3}}{2}}$$

(d) Find the exact value of $\sin^{-1}(\sin(3\pi/2))$.

$$= \sin^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) = \boxed{-\frac{\pi}{2}}$$

(REMEMBER THE RANGE OF $\sin^{-1}$ IS $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$)

11. Consider the function $f(x) = -4\cos\left(\frac{3}{2}(x + \pi/3)\right)$.

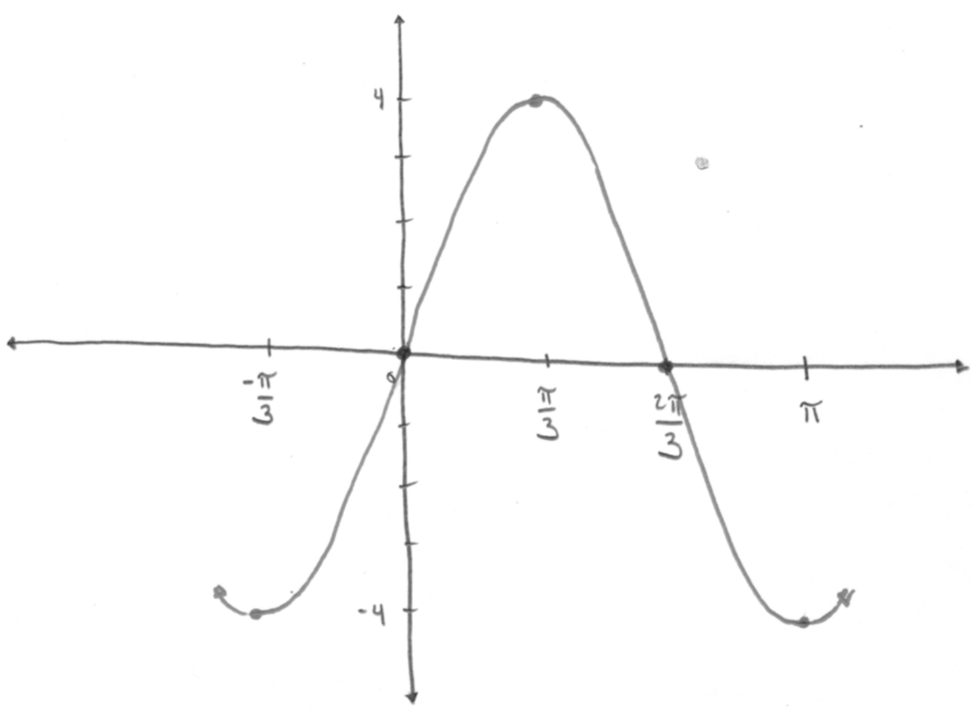
(a) What is the amplitude of f?

$$\boxed{4}$$

(b) What is the period of f?

$$\frac{2\pi}{\frac{3}{2}} = 2\pi \cdot \frac{2}{3} = \boxed{\frac{4\pi}{3}}$$

(c) Sketch the graph of f.



12. (a) Find the minimum/maximum value of the following quadratic function, and state whether it is a minimum or maximum value.

$$q(x) = -2x^2 + 4x - 11$$

LEAD COEFF $< 0 \Rightarrow$ MAXIMUM

MAXIMUM VALUE $= q\left(-\frac{b}{2a}\right) = q\left(-\frac{4}{2(-2)}\right)$

$= q(1) = -2 + 4 - 11 = \boxed{-9}$ MAX VALUE

ALT: $q(x) = -2(x-1)^2 - 9$

MAX VALUE

(b) Factor the following expression completely.

$$12x^5 + 12x^4 - 27x^3 - 27x^2$$

$3x^2(4x^3 + 4x^2 - 9x - 9)$

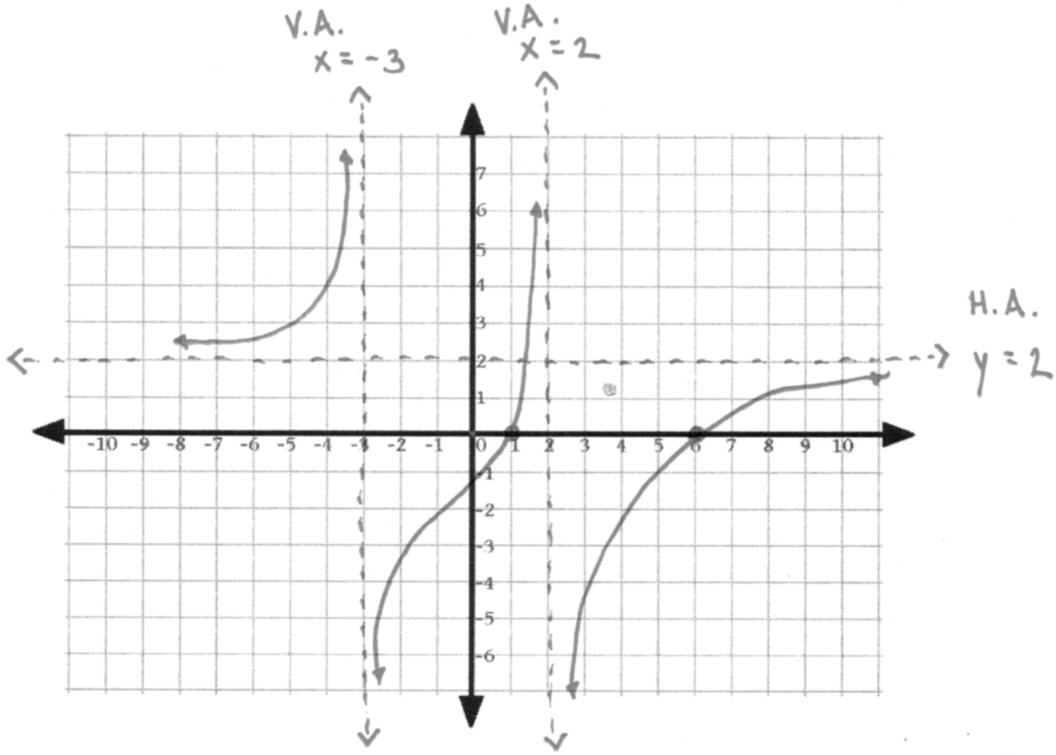
$3x^2(4x^2(x+1) - 9(x+1))$

$3x^2(4x^2-9)(x+1)$

$\boxed{3x^2(2x+3)(2x-3)(x+1)}$

7. Sketch the graph of the following rational function on the coordinate plane below. Label all x-intercepts and y-intercepts. Be sure that it is clear on which intervals the function is positive/negative, and label all vertical/horizontal asymptotes with their equations.

$$r(x) = \frac{2x^2 - 14x + 12}{x^2 + x - 6}$$



$$r(x) = \frac{2(x-1)(x-6)}{(x+3)(x-2)}$$

	-3	1	2	6	
$2(x-1)$	−	−	+	+	+
$(x-6)$	−	−	−	−	+
$(x+3)$	−	+	+	+	+
$(x-2)$	−	−	−	+	+
$\frac{2(x-1)(x-6)}{(x+3)(x-2)}$	+	−	+	−	+