MATH 195

§1.5 EQUATIONS

JOHN ADAMSKI

6/10/2016

21, 28, 33, 43, 45, 53, 55, 67, 77, 85, 87, 89, 97, 113

21. $2(1-x) = 3(1+2x) + 5$

$2 - 2x = 3 + 6x + 5$

$2 - 2x = 8 + 6x$

$-8 + 2x \quad -8 + 2x$

$\frac{-6}{8} = \frac{8x}{8}$

$x = -\frac{6}{8} = \boxed{-\frac{3}{4}}$

28. $\frac{4}{x-1} + \frac{2}{x+1} = \frac{35}{x^2-1}$ $\qquad$ LCD $= (x+1)(x-1)$

$\uparrow$

$(x+1)(x-1)$

CLEAR DENOMINATORS BY MULTIPLYING ALL TERMS BY LCD

$\frac{4}{x-1}(x+1)(x-1) + \frac{2}{x+1}(x+1)(x-1) = \frac{35}{(x+1)(x-1)}(x+1)(x-1)$

$4(x+1) + 2(x-1) = 35$

$4x + 4 + 2x - 2 = 35$

$6x + 2 = 35$

$6x = 33$

$x = \frac{33}{6} = \boxed{\frac{11}{2}}$

33. $P = 2l + 2w$ SOLVE FOR w

$P - 2l = 2w$

$$\frac{1}{2}(P - 2l) = w$$

43. $h = \frac{1}{2}gt^2 + v_0 t$ SOLVE FOR t.

$0 = \frac{1}{2}gt^2 + v_0 t - h$

THIS IS A QUADRATIC EQUATION IN t WITH COEFFICIENTS

$a = \frac{1}{2}g$, $b = v_0$, $c = -h$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}g\right)(-h)}}{2\left(\frac{1}{2}g\right)}$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 + 2gh}}{g}$$

45. $x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x + 4 = 0$ $\quad$ $x - 3 = 0$

$x = -4$, $\quad x = 3$

53. $2x^2 = 8$

$2x^2 - 8 = 0$

$2(x^2 - 4) = 0$

$2(x + 2)(x - 2) = 0$

$2 = 0$ $\quad$ $x + 2 = 0$ $\quad$ $x - 2 = 0$

$\emptyset$ $\quad$ $x = -2$ $\quad$ $x = 2$

55. $(2x-5)^2 = 81$

$\sqrt{(2x-5)^2} = \sqrt{81}$

$|2x-5| = 9$

$2x-5 = \pm 9$

$2x = 5 \pm 9$

$x = \frac{5 \pm 9}{2} = \frac{14}{2}, \frac{-4}{2} = \boxed{7, -2}$

67. $x^2 - 13x + 42 = 0$

$(x-7)(x-6) = 0$

$x-7=0 \quad x-6=0$

$\boxed{x = 7, \quad x = 6}$

BEGINNING THE METHOD OF SPLITTING THE MIDDLE TERM:

77. $7x^2 - 2x + 4 = 0$ $\quad \_\cdot\_ = 7 \cdot 4 = 28$

$\_+\_ = -2$

SINCE NO INTEGERS SATISFY THIS, WE RESORT TO USING THE QUADRATIC FORMULA

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(4)}}{2(7)}$

$= \frac{2 \pm \sqrt{4-112}}{14}$

OBSERVE THAT THE DISCRIMINANT IS NEGATIVE. HENCE NO (REAL) SOLUTIONS.

85. $4x^2 + 5x + \frac{13}{8} = 0$

DISCRIMINANT $= b^2 - 4ac = (5)^2 - 4(4)\left(\frac{13}{8}\right)$

$= 25 - \frac{4 \cdot 4 \cdot 13}{8} = 25 - 26 < 0$

SINCE THE DISCRIMINANT IS NEGATIVE, THIS EQUATION HAS 0 REAL SOLUTIONS

87. $\frac{x^2}{x+100} = 50$ CLEAR DENOMINATORS (CROSS CANCEL)

$x^2 = 50(x+100) = 50x + 5000$

$x^2 - 50x - 5000 = 0$

$(x-100)(x+50) = 0$

$\boxed{x = 100, \quad x = -50}$

89. $\frac{1}{x-1} + \frac{1}{x+2} = \frac{5}{4}$ LCD $= 4(x-1)(x+2)$

$\frac{1}{x-1} \cdot 4(x-1)(x+2) + \frac{1}{x+2} \cdot 4(x-1)(x+2) = \frac{5}{4} \cdot 4(x-1)(x+2)$

$4(x+2) + 4(x-1) = 5(x-1)(x+2)$

$4x + 8 + 4x - 4 = 5x^2 + 5x - 10$

$0 = 5x^2 - 3x - 14 = (5x+7)(x-2)$

$5x + 7 = 0$ $\qquad x - 2 = 0$

$5x = -7$ $\qquad x = 2$

$\boxed{x = \frac{-7}{5}, \quad}$

97. $\sqrt{2x+1} + 1 = x$

$\sqrt{2x+1} = x - 1$

$2x+1 = (x-1)^2 = x^2 - 2x + 1$

$0 = x^2 - 4x = x(x-4)$

$\boxed{x = 0, 4}$

113. $|3x+5| = 1$

$3x+5 = \pm 1$

$3x = -5 \pm 1$

$x = \dfrac{-5 \pm 1}{3} = \boxed{-\dfrac{4}{3}, -2}$