MATH 195

§4.5 EXPONENTIAL & LOGARITHMIC EQUATIONS

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1-3, 7, 17, 21, 27, 31, 37, 46, 49, 52, 55, 59, 61, 63

1. $e^x = 25$

$x = \ln 25 \approx 3.2189$

2. $\log(3x-6) = \log(x)$

$3x - 6 = x$

$2x = 6$

$x = 3$

3. $5^{x-1} = 125$

$5^{x-1} = 5^3$

$\downarrow$

$x - 1 = 3$

$\boxed{x = 4}$

7. $7^{2x-3} = 7^{6+5x}$

$\downarrow$

$2x - 3 = 6 + 5x$

$-9 = 3x$

$\boxed{x = -3}$

17. $3e^x = 10$

$e^x = \frac{10}{3}$

$\boxed{x = \ln \frac{10}{3}}$

21. $e^{1-4x} = 2$

$1 - 4x = \ln 2$

$\boxed{\frac{1 - \ln 2}{4} = x}$

27. $4(1 + 10^{5x}) = 9$

$1 + 10^{5x} = \frac{9}{4}$

$10^{5x} = \frac{9}{4} - 1 = \frac{5}{4}$

$5x = \log \frac{5}{4}$

$\boxed{x = \frac{1}{5} \log \frac{5}{4}}$

31. $4^x + 2^{1+2x} = 50$

$(2^2)^x + 2^{1+2x} = 50$

$2^{2x} + 2^{2x+1} = 50$

TRICKY! BUT YOU CAN HANDLE IT!

$2^{2x}(1 + 2) = 50$

$2^{2x} = \frac{50}{3}$

$2x \ln 2 = \ln \frac{50}{3}$

$$x = \frac{\ln \frac{50}{3}}{2 \ln 2} \quad \text{or} \quad \frac{1}{2} \log_2 \frac{50}{3}$$

SAME BY CHANGE OF BASE FORMULA

37. $\frac{50}{1 + e^{-x}} = 4$

$\frac{1 + e^{-x}}{50} = \frac{1}{4}$

$1 + e^{-x} = \frac{50}{4} = \frac{25}{2}$

$e^{-x} = \frac{23}{2}$

$-x = \ln\left(\frac{23}{2}\right)$

SEE WHAT I DID?

$$x = \ln\left(\frac{2}{23}\right)$$

46. $x^2 10^x - x 10^x = 2(10^x)$

$10^x(x^2 - x - 2) = 0$

$10^x(x-2)(x+1) = 0$

$10^x = 0$ $\quad$ $x - 2 = 0$ $\quad$ $x + 1 = 0$

NO SOLNS $\quad$ $\boxed{x = 2 \qquad x = -1}$

49. $\log x + \log(x-1) = \log(4x)$

$\log(x^2 - x) = \log(4x)$

$x^2 - x = 4x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$ $\quad$ $\boxed{x = 3}$

REJECT! $\log(0)$ IS UNDEFINED

52. $\ln\left(x - \frac{1}{2}\right) + \ln(2) = 2\ln(x)$

$\ln(2x - 1) = \ln(x^2)$

$2x - 1 = x^2 \rightarrow 0 = x^2 - 2x + 1$

$0 = (x-1)^2$

$\boxed{x = 1}$

55. $\ln x = 10$

$\boxed{x = e^{10}}$

59. $\log(3x + 5) = 2$

$3x + 5 = 10^2 = 100$

$3x = 95$

$\boxed{x = \frac{95}{3}}$

61. $4 - \log(3 - x) = 3$

$1 = \log(3 - x)$

$10 = 3 - x$

$\boxed{x = -7}$

63. $\log_2 x + \log_2(x - 3) = 2$

$\log_2(x^2 - 3x) = 2$

$x^2 - 3x = 2^2 = 4$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$\boxed{x = 4}$, $x = -1$ ← REJECT!