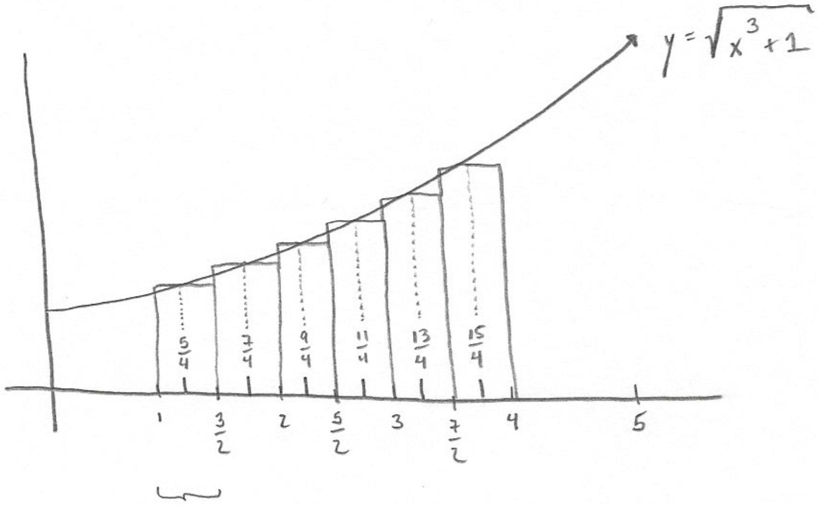
Name: *ANSWER KEY* Due 12/19/2016

Math 201-EC Exam 4

Directions Answer all questions in the space provided. Show all work and box your final answers. Turn in this exam in NAC 0/201 on Monday, 12/19/2016, before we take our final exam at 1pm. (Please arrive 15 minutes early.) Good luck!

1. Use the Midpoint Rule with $n = 6$ to approximate the following integral. You may leave your answer as a sum of terms.

$$\int_1^4 \sqrt{x^3+1}\,dx$$



$$\Delta x = \frac{b-a}{n} = \frac{4-1}{6} = \frac{1}{2} \qquad x_i = a + i\Delta x = 1 + i \cdot \frac{1}{2}$$

$$x_i^* = \frac{x_{i-1} + x_i}{2}$$

$$M_6 = \sum_{i=1}^{6} f(x_i^*)\,\Delta x$$

$$M_6 = \left[f\left(\tfrac{5}{4}\right) + f\left(\tfrac{7}{4}\right) + f\left(\tfrac{9}{4}\right) + f\left(\tfrac{11}{4}\right) + f\left(\tfrac{13}{4}\right) + f\left(\tfrac{15}{4}\right) \right]\left(\tfrac{1}{2}\right)$$

$$\boxed{M_6 = \left[\sqrt{\left(\tfrac{5}{4}\right)^3+1} + \sqrt{\left(\tfrac{7}{4}\right)^3+1} + \sqrt{\left(\tfrac{9}{4}\right)^3+1} + \sqrt{\left(\tfrac{11}{4}\right)^3+1} + \sqrt{\left(\tfrac{13}{4}\right)^3+1} + \sqrt{\left(\tfrac{15}{4}\right)^3+1} \right]\left(\tfrac{1}{2}\right)}$$

2. Evaluate each of the following definite integrals.

(a) $\int_0^1 x^2(\sqrt[3]{x} + \sqrt[4]{x})\, dx$

$$= \int_0^1 x^2\left(x^{1/3} + x^{1/4}\right) dx = \int_0^1 x^{7/3} + x^{9/4}\, dx$$

$$= \left(\frac{3}{10} x^{10/3} + \frac{4}{13} x^{13/4}\right)\Bigg|_0^1$$

$$= \left(\frac{3}{10} + \frac{4}{13}\right) - (0) = \frac{39 + 40}{130} = \boxed{\frac{79}{130}}$$

(b) $\int_{1/2}^1 \frac{\cos(1/x^2)}{x^3}\, dx \quad = \int_{1/2}^{1} \cos\left(x^{-2}\right) x^{-3}\, dx$

LET $u = x^{-2}$

$du = -2x^{-3}\, dx$

$-\frac{1}{2}\, du = x^{-3}\, dx$

$$\int_{x=1/2}^{x=1} \leadsto \int_{u=4}^{u=1}$$

$$\leadsto -\frac{1}{2}\int_4^1 \cos(u)\, du = -\frac{1}{2}\sin(u)\Big|_4^1 = -\frac{1}{2}\left[\sin(1) - \sin(4)\right]$$

$$= \boxed{\frac{1}{2}\left(\sin(4) - \sin(1)\right)}$$

3. Evaluate each of the following indefinite integrals.

(a) $\displaystyle\int \frac{x^2}{\sqrt[3]{1+x^3}}\,dx$

LET $u = 1 + x^3$

$du = 3x^2 dx \longrightarrow \frac{1}{3} du = x^2 dx$

$\rightsquigarrow \frac{1}{3} \int \frac{1}{\sqrt[3]{u}}\, du = \frac{1}{3} \cdot \frac{3}{2} u^{2/3} + C$

$\rightsquigarrow \boxed{\frac{1}{2} (1 + x^3)^{2/3} + C}$

(b) $\displaystyle\int \underbrace{(\tan^2 5\theta + 1)}_{= \sec^2 5\theta} \sec 5\theta \tan 5\theta \, d\theta$

$\int \sec^2 5\theta \sec 5\theta \tan 5\theta \, d\theta$

LET $u = \sec 5\theta$

$du = 5 \sec\theta \tan\theta \, d\theta$

$\frac{1}{5} du = \sec\theta \tan\theta \, d\theta$

$\rightsquigarrow \frac{1}{5} \int u^2 \, du = \frac{1}{15} u^3 + C$

$\rightsquigarrow = \boxed{\frac{1}{15} \sec^3 5\theta + C}$

4. Let $F(x) = \int_{\cos x}^{\sin^2 x} (1+t^2)^{10}\, dt$. Find $F'(x)$.
Hint: Use the Fundamental Theorem of Calculus (part I).

$$F(x) = \int_{\cos x}^{0} (1+t^2)^{10}\, dt + \int_{0}^{\sin^2 x} (1+t^2)^{10}\, dt$$

$$= -\int_{0}^{\cos x} (1+t^2)^{10}\, dt + \int_{0}^{\sin^2 x} (1+t^2)^{10}\, dt$$

$$F'(x) = \boxed{(1+\cos^2 x)^{10} \sin x + (1+\sin^4 t)^{10} \cdot 2 \sin x \cos x}$$

5. Suppose $f(6) = 5$ and $f'(x) \leq 4$ for all x. Find the largest possible value of $f(10)$.
Hint: Use the Mean Value Theorem.

THERE EXIST c, $6 < c < 10$, SUCH THAT $f'(c) = \dfrac{f(10) - f(6)}{10 - 6}$

i.e. $f(10) = 4 f'(c) + f(6)$

$f(10) \leq 4(4) + 5 = 21$

$$\boxed{f(10) \leq 21}$$