Name: *ANSWER KEY* **9/29/2016**

Math 201-EC Exam 1

Directions Answer all questions in the space provided. Show all work and box your final answers. Answers with no work shown will not receive full credit. If you have any questions, just ask. Good luck!

1. (8 points) Use interval notation to describe the domain of the following function.

$$f(x) = \frac{\sqrt{x+5}}{2x^2+3x-9}$$

TWO THINGS : (1) $x+5 \geq 0$ (NON-NEG. RADICAND)

$x \geq -5$

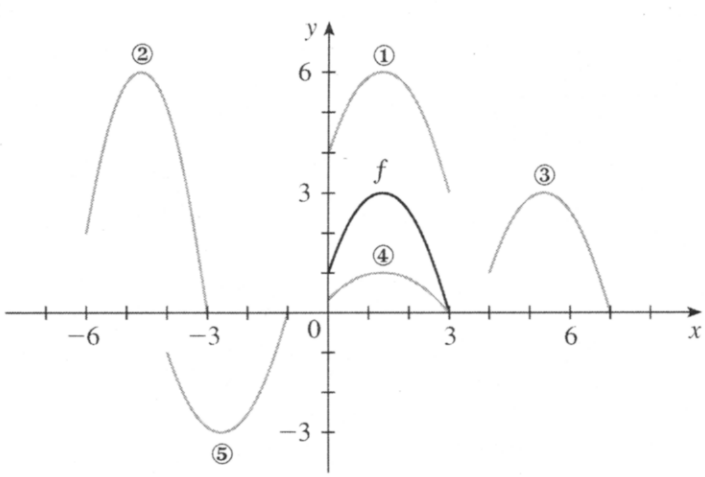
(2) $2x^2+3x-9 \neq 0$ (NON-ZERO DENOMINATOR)

$(2x-3)(x+3) \neq 0$

$x \neq \frac{3}{2}$ $\qquad$ $x \neq -3$

COMBINE PART (1) AND (2) : $\boxed{[-5,-3) \cup (-3,\tfrac{3}{2}) \cup (\tfrac{3}{2}, \infty)}$

2. (8 points) The graph of $y = f(x)$ is given below. Match each equation with its graph.



(a) $y = f(x-4)$ ③

(b) $y = f(x) + 3$ ①

(c) $y = \frac{1}{3}f(x)$ ④

(d) $y = -f(x+4)$ ⑤

(e) $y = 2f(x+6)$ ②

3. Suppose f and g are functions defined by

$$f(x) = x + \frac{1}{x} \quad \text{and} \quad g(x) = \frac{x+1}{x+2}.$$

(a) (4 points) Find $f \circ g(x)$ (no need to simplify) and state its domain.

$f(g(x)) = g(x) + \frac{1}{g(x)} = \boxed{\frac{x+1}{x+2} + \frac{x+2}{x+1}}$, DOM: $\boxed{x \neq -1, -2}$

(b) (4 points) Find $g \circ f(x)$ (no need to simplify) and state its domain.

$g(f(x)) = \frac{f(x)+1}{f(x)+2} = \boxed{\frac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2}}$,

DOM: $\boxed{x \neq 0 \text{ AND}}$

$x + \frac{1}{x} + 2 \neq 0$

$x^2 + 2x + 1 \neq 0$

$(x+1)^2 \neq 0$

$\boxed{x \neq -1}$

(c) (4 points) Find $f \circ f(x)$ (no need to simplify) and state its domain.

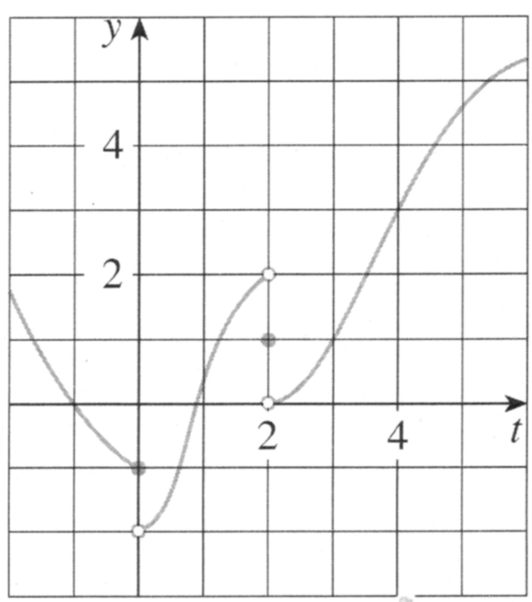
$f(f(x)) = f(x) + \frac{1}{f(x)} = \boxed{x + \frac{1}{x} + \frac{1}{1 + \frac{1}{x}}}$,

DOM: $\boxed{x \neq 0 \text{ AND}}$

$1 + \frac{1}{x} \neq 0 \Rightarrow x + 1 \neq 0$

$\boxed{x \neq -1}$

4. (8 points) Consider the function g whose graph is shown below.



State the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{t\to 0^-} g(t)$ -1

(b) $\lim_{t\to 0^+} g(t)$ -2

(c) $\lim_{t\to 0} g(t)$ DNE BECAUSE $\lim_{t\to 0^-} g(t) \neq \lim_{t\to 0^+} g(t)$

(d) $\lim_{t\to 2^-} g(t)$ 2

(e) $\lim_{t\to 2^+} g(t)$ 0

(f) $\lim_{t\to 2} g(t)$ DNE BECAUSE $\lim_{t\to 2^-} g(t) \neq \lim_{t\to 2^+} g(t)$

5. Evaluate the following limits.

(a) (8 points) $\lim_{t\to -4} \frac{\frac{1}{4}+\frac{1}{t}}{4+t}$ $= \lim_{t\to -4} \frac{\frac{t+4}{4t}}{4+t}$

$= \lim_{t\to -4} \frac{\cancel{t+4}}{4t\,(\cancel{4+t})} = \lim_{t\to -4} \frac{1}{4t} = \boxed{-\frac{1}{16}}$

(b) (8 points) $\lim_{\theta\to 0} \frac{\sin(2\theta)}{\sin(5\theta)}$ $= \lim_{\theta\to 0} \frac{\frac{\sin(2\theta)}{10\theta}}{\frac{\sin(5\theta)}{10\theta}}$

$= \lim_{\theta\to 0} \frac{\frac{1}{5}\cdot\frac{\sin(2\theta)}{2\theta}}{\frac{1}{2}\cdot\frac{\sin(5\theta)}{5\theta}} = \frac{\frac{1}{5}\cdot\lim_{\theta\to 0}\frac{\sin(2\theta)}{2\theta}}{\frac{1}{2}\cdot\lim_{\theta\to 0}\frac{\sin(5\theta)}{5\theta}}$

$= \frac{\frac{1}{5}\cdot 1}{\frac{1}{2}\cdot 1} = \boxed{\frac{2}{5}}$

8. Evaluate the following limits.

(a) (4 points) $\lim_{x\to\infty} \frac{x^3+5x}{2x^3-x^2+4}$ $= \lim_{x\to\infty} \frac{\cancel{x^3}\left(1+\frac{5}{x^2}\right)}{\cancel{x^3}\left(2-\frac{1}{x}+\frac{4}{x^3}\right)}$

$$= \frac{1+\lim_{x\to\infty}\frac{5}{x^2}}{2-\lim_{x\to\infty}\frac{1}{x}+\lim_{x\to\infty}\frac{4}{x^3}} = \frac{1+0}{2-0+0} = \boxed{\frac{1}{2}}$$

(b) (4 points) $\lim_{x\to-\infty} \frac{7-x}{2x^2-1}$ $= \lim_{x\to-\infty} \frac{\cancel{x^2}\left(\frac{7}{x}-\frac{1}{x}\right)}{\cancel{x^2}\left(2-\frac{1}{x^2}\right)}$

$$= \frac{\lim_{x\to-\infty}\frac{7}{x}-\lim_{x\to-\infty}\frac{1}{x}}{2-\lim_{x\to-\infty}\frac{1}{x^2}} = \frac{0-0}{2-0} = \boxed{0}$$

6. (8 points) Use the Squeeze Theorem to evaluate the following limit.

$$\lim_{x\to 0} x^4 \cos(\frac{2\pi}{x})$$

FOR $x \neq 0$, WE HAVE

$$-1 \leq \cos\left(\frac{2\pi}{x}\right) \leq 1$$

$$-x^4 \leq x^4 \cos\left(\frac{2\pi}{x}\right) \leq x^4$$

AND SO $\underbrace{\lim_{x\to 0} -x^4}_{0} \leq \lim_{x\to 0} x^4 \cos\left(\frac{2\pi}{x}\right) \leq \underbrace{\lim_{x\to 0} x^4}_{0}$

THEREFORE, BY THE SQUEEZE THM, $\lim_{x\to 0} x^4 \cos\left(\frac{2\pi}{x}\right) = \boxed{0}$

7. (8 points) For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

THIS PIECEWISE DEFINED FUNCTION IS COMPOSED OF POLYNOMIALS, EACH OF WHICH IS CONTINUOUS ON ITS DOMAIN. WE JUST NEED TO FIND c SUCH THAT $f(x)$ IS CONTINUOUS AT 2. THAT IS:

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$

$$\Rightarrow \underbrace{\lim_{x\to 2^-} cx^2 + 2x}_{4c+4} = \underbrace{\lim_{x\to 2^+} x^3 - cx}_{8-2c} = 8 - 2c$$

$$4c + 4 = 8 - 2c$$

$$6c = 4$$

$$\boxed{c = \frac{2}{3}}$$