Name: *ANSWER KEY* 12/1/2016

Math 201-EC Exam 3

Directions Answer all questions in the space provided. Show all work and box your final answers. Answers with no work shown will not receive full credit. If you have any questions, just ask. Good luck!

1. (8 points) Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1, \quad -\frac{1}{2} \leq x \leq 4.$$

CRITICAL POINTS: $f'(x) = 3x^2 - 6x = 3x(x-2) = 0$

$x = 0, 2$

END POINTS : $x = -\frac{1}{2}, 4$

x	$f(x)$
$-\frac{1}{2}$	$(-\frac{1}{2})^3 - 3(-\frac{1}{2})^2 + 1 = -\frac{1}{8} - \frac{3}{4} + 1 = \frac{-1-6+8}{8} = \frac{1}{8}$
0	$(0)^3 - 3(0)^2 + 1 = 1$
2	$(2)^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3$
4	$(4)^3 - 3(4)^2 + 1 = 64 - 48 + 1 = 17$

ABSOLUTE MAXIMUM VALUE 17

ABSOLUTE MINIMUM VALUE -3

2. (8 points) Suppose that $f(4) = 12$ and $4 \leq f'(x) \leq 5$ for all x. What are the minimum and maximum possible values of $f(7)$? Hint: Use the Mean Value Theorem.

MVT $\Rightarrow \exists c \in (4,7)$ SUCH THAT $f(7) - f(4) = f'(c)(7-4)$

THAT IS, $f(7) = f(4) + f'(c)(7-4) = 12 + 3f'(c)$

SINCE $4 \leq f'(c) \leq 5$

$12 \leq 3f'(c) \leq 15$

$24 \leq \underbrace{12 + 3f'(c)}_{f(7)} \leq 27$

MINIMUM 24

MAXIMUM 27

3. Consider the following function.

$$f(x) = x\sqrt{2 - x^2}$$

(a) (2 points) What is the domain of f.

$2 - x^2 \geq 0$

$x^2 \leq 2$

$|x| \leq \sqrt{2}$

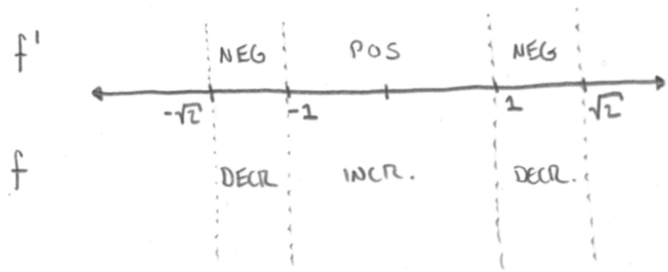
$\boxed{[-\sqrt{2}, \sqrt{2}]}$

(b) (4 points) On what intervals is f increasing/decreasing?

$$f'(x) = \sqrt{2-x^2} + x \cdot \frac{1}{\cancel{2}} (2-x^2)^{-1/2} (-\cancel{2}x)$$

$$= \frac{1}{\sqrt{2-x^2}} \left((2-x^2) - x^2 \right)$$

$$f'(x) = \frac{2-2x^2}{\sqrt{2-x^2}} = 0 \quad \text{WHEN } x = \pm 1$$



$\boxed{\text{INCREASING ON } (-1, 1) \quad \text{DECREASING ON } (-\sqrt{2}, -1) \cup (1, \sqrt{2})}$

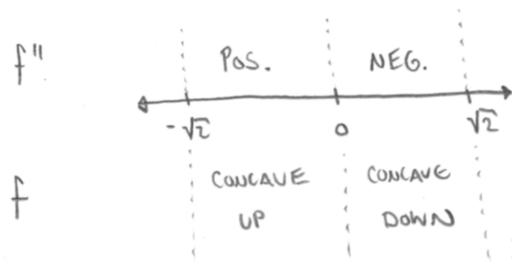
(c) (4 points) On what intervals in f concave up/down?

$$f''(x) = \frac{\sqrt{2-x^2}\,(-4x) - (2-2x^2) \cdot \frac{1}{\cancel{2}}(2-x^2)^{-1/2}(-\cancel{2}x)}{2-x^2}$$

$$= \frac{(2-x^2)^{-1/2}\left[-4x(2-x^2) + x(2-2x^2)\right]}{2-x^2}$$

NOT IN DOMAIN

$$= \frac{2x^3 - 6x}{(2-x^2)^{3/2}} = \frac{2x(x^2-3)}{(2-x^2)^{3/2}} = 0 \quad \text{WHEN } x = 0, \pm\sqrt{3}$$



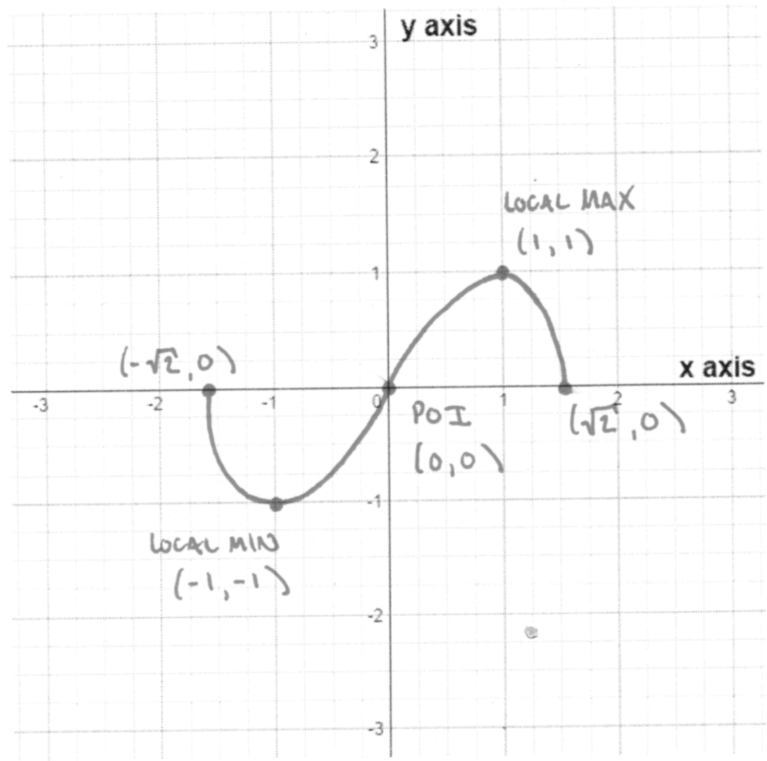
$\boxed{\text{CONCAVE UP ON } (-\sqrt{2}, 0) \quad \text{CONCAVE DOWN ON } (0, \sqrt{2})}$

(d) (4 points) Find the coordinates of all local maxima, local minima, and points of inflection.

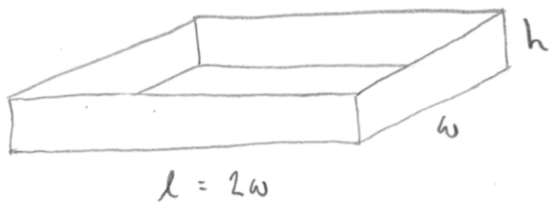
LOCAL MAX $(1, f(1)) = (1, 1)$ POI $(0, f(0)) = (0,0)$

LOCAL MIN $(-1, f(-1)) = (-1, -1)$

(e) (4 points) Sketch the graph of f below. Label the features you found in parts (b) though (d).



4. (8 points) A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.



$V = lwh$

$10 = 2w^2h \Rightarrow h = \frac{5}{w^2}$

COST OF BASE $= 10\,lw = 20w^2$

COST OF 4 SIDES $= 2(6wh) + 2(6lh)$

$= 12\cancel{w} \cdot \frac{5}{w^{\cancel{2}}} + 12 \cdot 2\cancel{w} \cdot \frac{5}{w^{\cancel{2}}} = \frac{60}{w} + \frac{120}{w}$

$= \frac{180}{w}$

TOTAL COST $C(w) = 20w^2 + \frac{180}{w}, \quad w > 0$

CRITICAL POINT(S): $C'(w) = 40w - \frac{180}{w^2} = 0$

$\rightarrow 40w = \frac{180}{w^2} \longrightarrow w^3 = \frac{9}{2} \longrightarrow w = \sqrt[3]{\frac{9}{2}}$

SECOND DERIV. TEST $C''(w) = 40 + \frac{2 \cdot 180}{3w^3} \Rightarrow C''\left(\sqrt[3]{\frac{9}{2}}\right) > 0 \Rightarrow$ MINIMUM!

IN FACT $C''(w) > 0$ FOR ALL $w > 0$, SO $y = C(x)$ IS CONCAVE UP & THIS MUST BE THE ABSOLUTE MINIMUM.

$C\left(\sqrt[3]{\frac{9}{2}}\right) = 20\left(\frac{9}{2}\right)^{2/3} + 180\left(\frac{2}{9}\right) = 40 + 20\left(\frac{9}{2}\right)^{2/3}$