Name: *ANSWER KEY* 11/3/2016

Math 201-EC Exam 2

Directions Answer all questions in the space provided. Show all work and box your final answers. Answers with no work shown will not receive full credit. If you have any questions, just ask. Good luck!

1. (a) (8 points) Use the definition of derivative (as a limit) to find the derivative of $f(x) = \frac{2x}{3x+4}$ at $x = 2$.

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \frac{\frac{2(2+h)}{3(2+h)+4} - \frac{2(2)}{3(2)+4}}{h}$$

$$= \lim_{h \to 0} \frac{(4+2h)10 - 4(10+3h)}{10(10+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{40} + 20h - \cancel{40} - 12h}{(100+10h)h}$$

$$= \lim_{h \to 0} \frac{8\cancel{h}}{(100+10h)\cancel{h}} = \frac{8}{100} = \boxed{\frac{2}{25}}$$

(b) (8 points) Use your answer from part (a) to write an equation of the tangent line to the curve $y = \frac{2x}{3x+4}$ at the point $(2, \frac{2}{5})$.

$$\boxed{y - \frac{2}{5} = \frac{2}{25}(x-2) \quad \text{or} \quad y = \frac{2}{25}x + \frac{6}{25}}$$

2. Differentiate the following functions.

(a) (8 points) $f(x) = 6x^3 - \sqrt[5]{x} - \dfrac{4}{x^2} + \sin x$

$$\boxed{f'(x) = 18x^2 - \frac{1}{5}x^{-\frac{4}{5}} + 8x^{-3} + \cos x}$$

(b) (8 points) $g(x) = x^6 \tan x$

$$g'(x) = \tan x \cdot \frac{d}{dx}[x^6] + x^6 \cdot \frac{d}{dx}[\tan x]$$

$$= \tan x \cdot 6x^5 + x^6 \cdot \sec^2 x$$

$$\boxed{g'(x) = 6x^5 \tan x + x^6 \sec^2 x}$$

(c) (8 points) $h(x) = \dfrac{x^6}{4 - 3x^5}$

$$h'(x) = \frac{(4-3x^5)\cdot\frac{d}{dx}[x^6] - x^6 \cdot \frac{d}{dx}[4-3x^5]}{(4-3x^5)^2}$$

$$= \frac{(4-3x^5)\cdot 6x^5 - x^6(-15x^4)}{(4-3x^5)^2}$$

$$\boxed{h'(x) = \frac{24x^5 - 18x^{10} + 15x^{10}}{(4-3x^5)^2} = \frac{24x^5 - 3x^{10}}{(4-3x^5)^2}}$$

3. Differentiate the following functions.

(a) (8 points) $f(x) = \cos\sqrt{x^2+1}$

$$f'(x) = -\sin\sqrt{x^2+1} \cdot \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x$$

$$\boxed{f'(x) = \frac{-x\sin\sqrt{x^2+1}}{\sqrt{x^2+1}}}$$

(b) (8 points) $g(x) = \left(\frac{2x+1}{1-3x}\right)^8$

$$\boxed{g'(x) = 8\left(\frac{2x+1}{1-3x}\right)^7 \cdot \frac{(1-3x)\cdot 2 - (2x+1)(-3)}{(1-3x)^2}}$$

(c) (8 points) $h(x) = \sec\left(\frac{1}{x}\right)$

NOTE: $\frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{\cos x \cdot 0 - 1(-\sin x)}{\cos^2 x}$

$= \frac{\sin x}{\cos^2 x} = \tan x \sec x$

$$h'(x) = \tan\left(\frac{1}{x}\right)\sec\left(\frac{1}{x}\right)\cdot(-x^{-2})$$

$$\boxed{h'(x) = \frac{-\tan\left(\frac{1}{x}\right)\sec\left(\frac{1}{x}\right)}{x^2}}$$

4. Consider the following implicit equation.

$$6x^3 + x^2y - xy^3 = 6 \tag{$\star$}$$

(a) (8 points) Use implicit differentiation to find $\dfrac{dy}{dx}$.

$$18x^2 + 2xy + x^2\frac{dy}{dx} - y^3 - x \cdot 3y^2\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} - 3xy^2\frac{dy}{dx} = y^3 - 18x^2 - 2xy$$

$$(x^2 - 3xy^2)\frac{dy}{dx} = y^3 - 18x^2 - 2xy$$

$$\boxed{\frac{dy}{dx} = \frac{y^3 - 18x^2 - 2xy}{x^2 - 3xy^2}}$$

(b) (8 points) Use your answer from part (a) to write an equation for the line tangent to the curve defined by equation ($\star$) at the point $(2, 3)$.

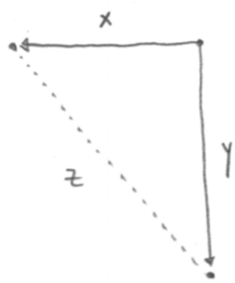
WHEN $x = 2$, $y = 3$ WE HAVE $\dfrac{dy}{dx} = \dfrac{(3)^3 - 18(2^2) - 2(2)(3)}{(2)^2 - 3(2)(3)^2}$

$$= \frac{27 - 72 - 12}{4 - 54} = \frac{-57}{-50} = \frac{57}{50}$$

(OR 1.14)

$$\therefore \quad \boxed{y - 3 = \frac{57}{50}(x - 2)}$$

5. (8 points) Two cars start moving from the same point. One travels south at 40 mi/h and the other travels west at 30 mi/h. At what rate is the distance between the cars increasing two hours later?



$$x^2 + y^2 = z^2$$

$$\Big\downarrow \frac{d}{dt}$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt} \qquad (*)$$

AFTER 2 HRS OF TRAVELLING AT 30 MPH & 40 MPH, x & y ARE 60 mi & 80 mi, RESPECTIVELY.

$$\left(\begin{array}{l} x = 60 \\ y = 80 \end{array} \right)$$

$z^2 = 60^2 + 80^2 = 100^2 \longrightarrow \underline{\underline{z = 100}}$ ALSO $\frac{dx}{dt} = 30$

$\frac{dy}{dt} = 40$

$$\therefore \quad (*) = (60)(30) + (80)(40) = (100)\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{(60)(30) + (80)(40)}{(100)} = \frac{1800 + 3200}{100} = \frac{5000}{100} = \boxed{50 \text{ MPH}}$$